

# Coherence of Multi-Dimensional Pair Production Discharges in Polar Caps of Pulsars

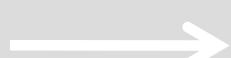
Chernoglazov A., Philippov A., Timokhin A. 2024

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(Shunshun Cao)

2024.10

27 pages in total

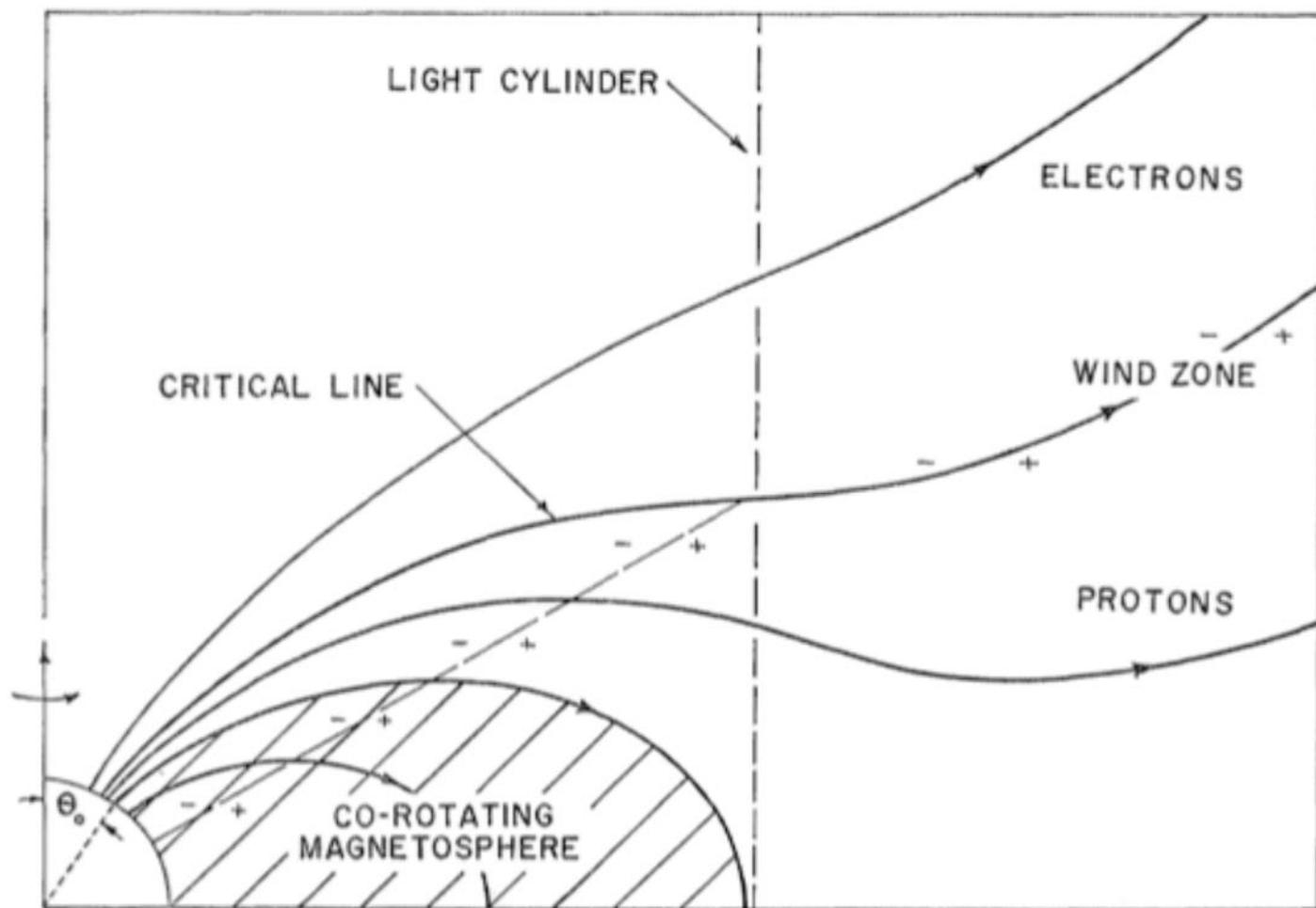
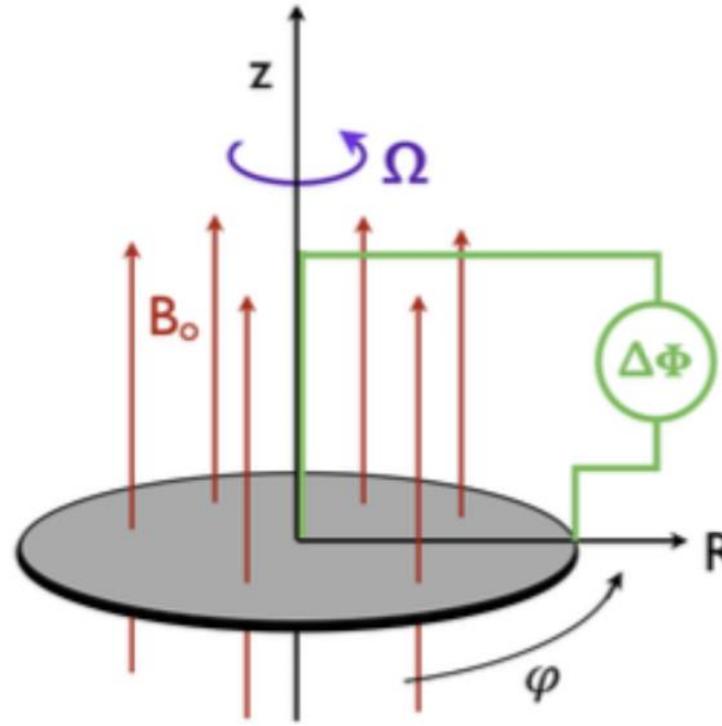


# I. Introduction

## (i) Pulsar magnetosphere and polar cap



A note in Zhihu  
for Amato's paper.

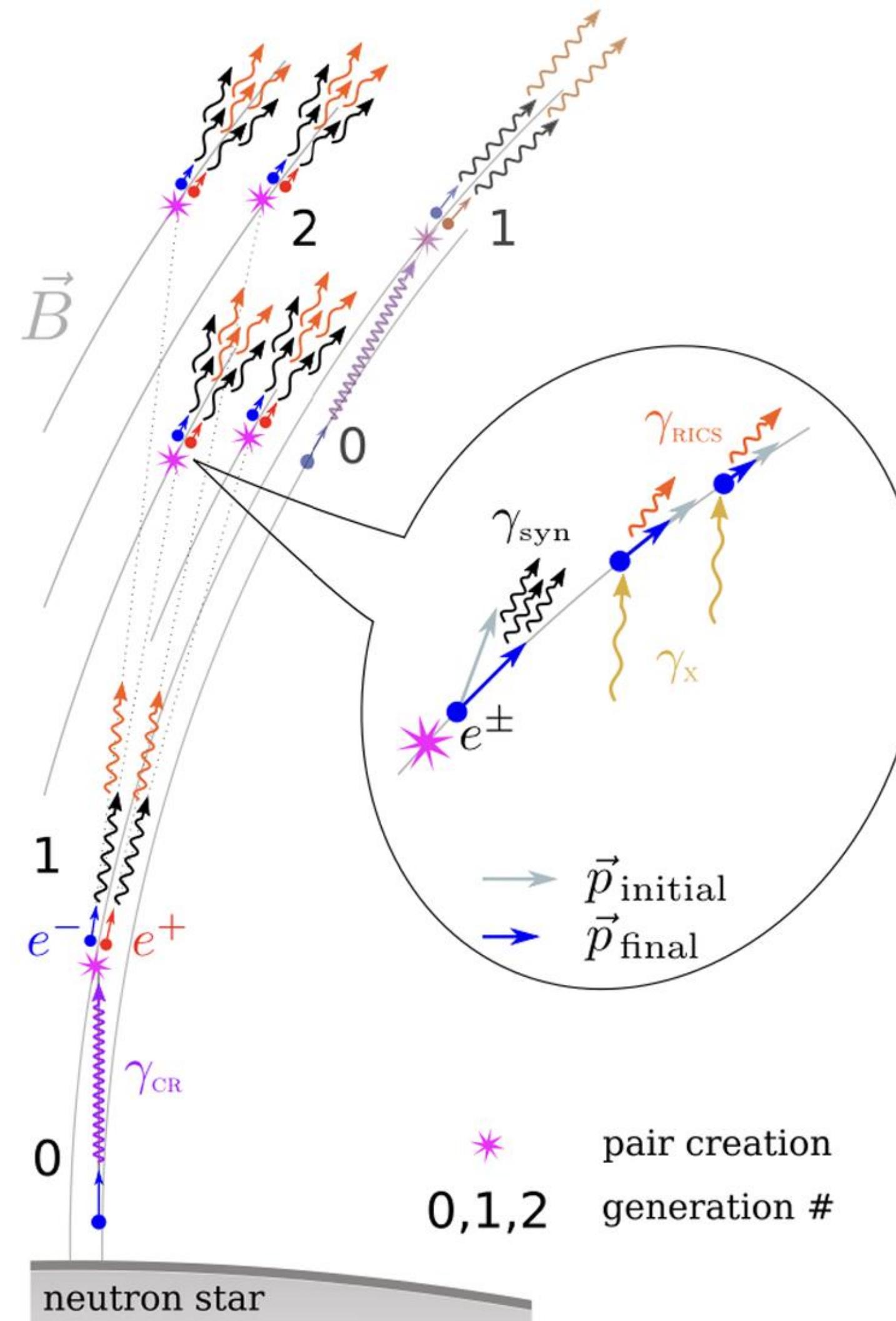
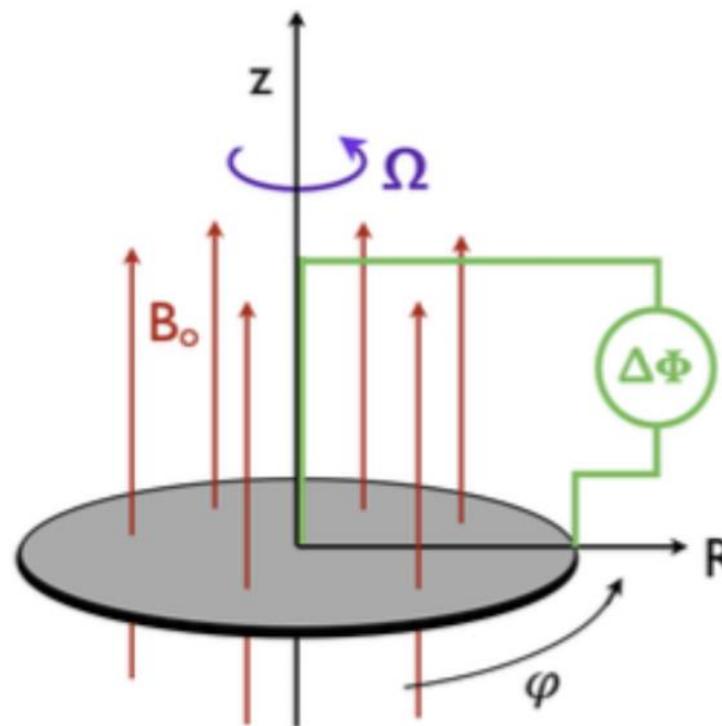
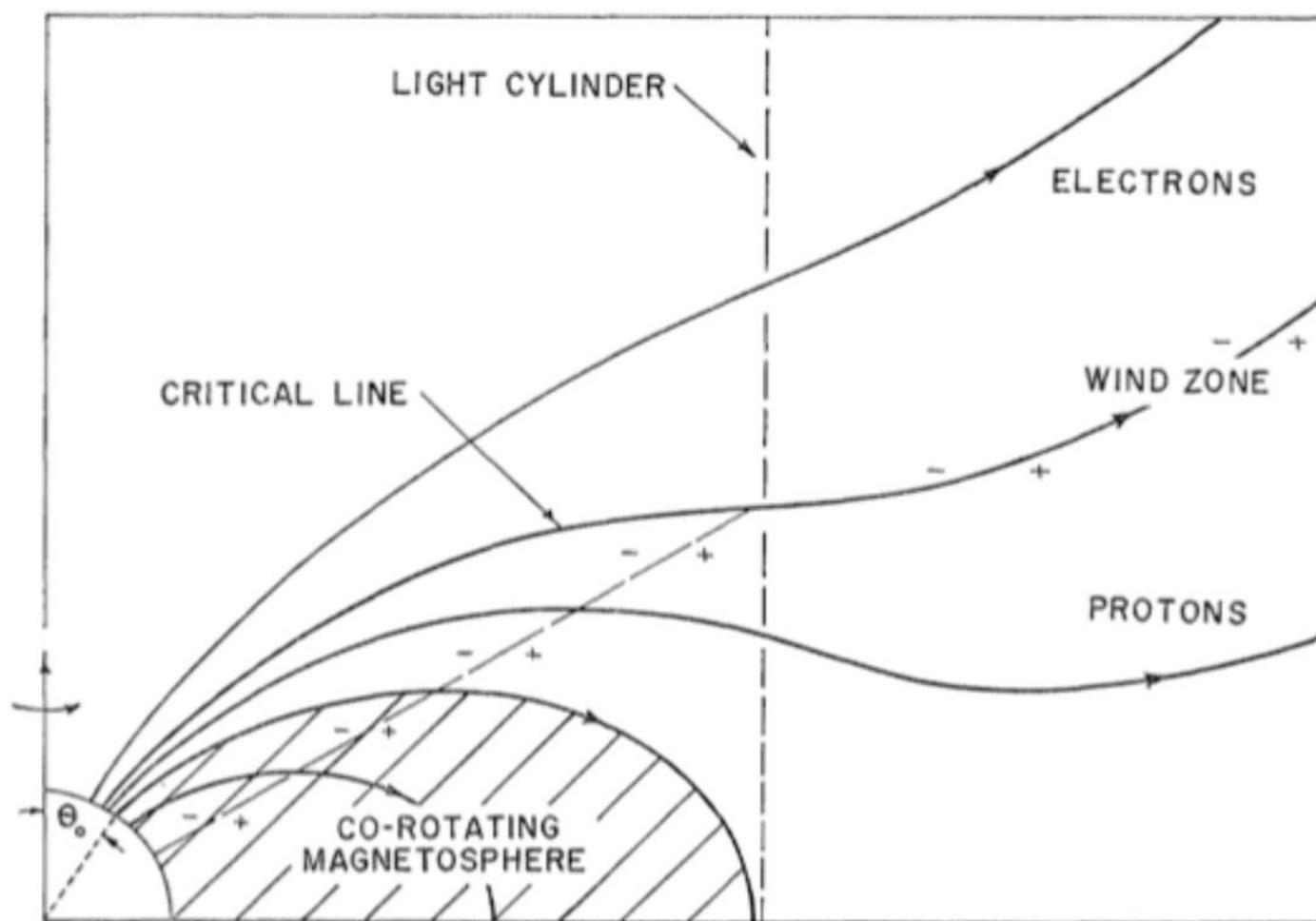


Pulsar  $\approx$  Faraday Disk  
Rotating compact object in magnetic field  
→ Electric field distribution  
→→ Provide acceleration regions

Amato 2024 arxiv.



A note in Zhihu  
for Amato's paper.



Initial particles in  $E$  field  
 → Accelerated particles  
 →→ Emitted photons  
 (curvature or ICS)  
 →→→ Pair ( $e^\pm$ ) creation  
 →→→ Charge separation  
 screen original  $E$  field

(Discharge process)

Amato 2024 arxiv.

Charged particles fill the pulsar surroundings  $\rightarrow$  magnetized plasma  $\rightarrow$  magnetosphere

Static magnetosphere:

$$q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) = 0$$

Corotation condition:

$$\mathbf{E} + (\boldsymbol{\Omega} \times \mathbf{r}) \times \mathbf{B} = 0$$

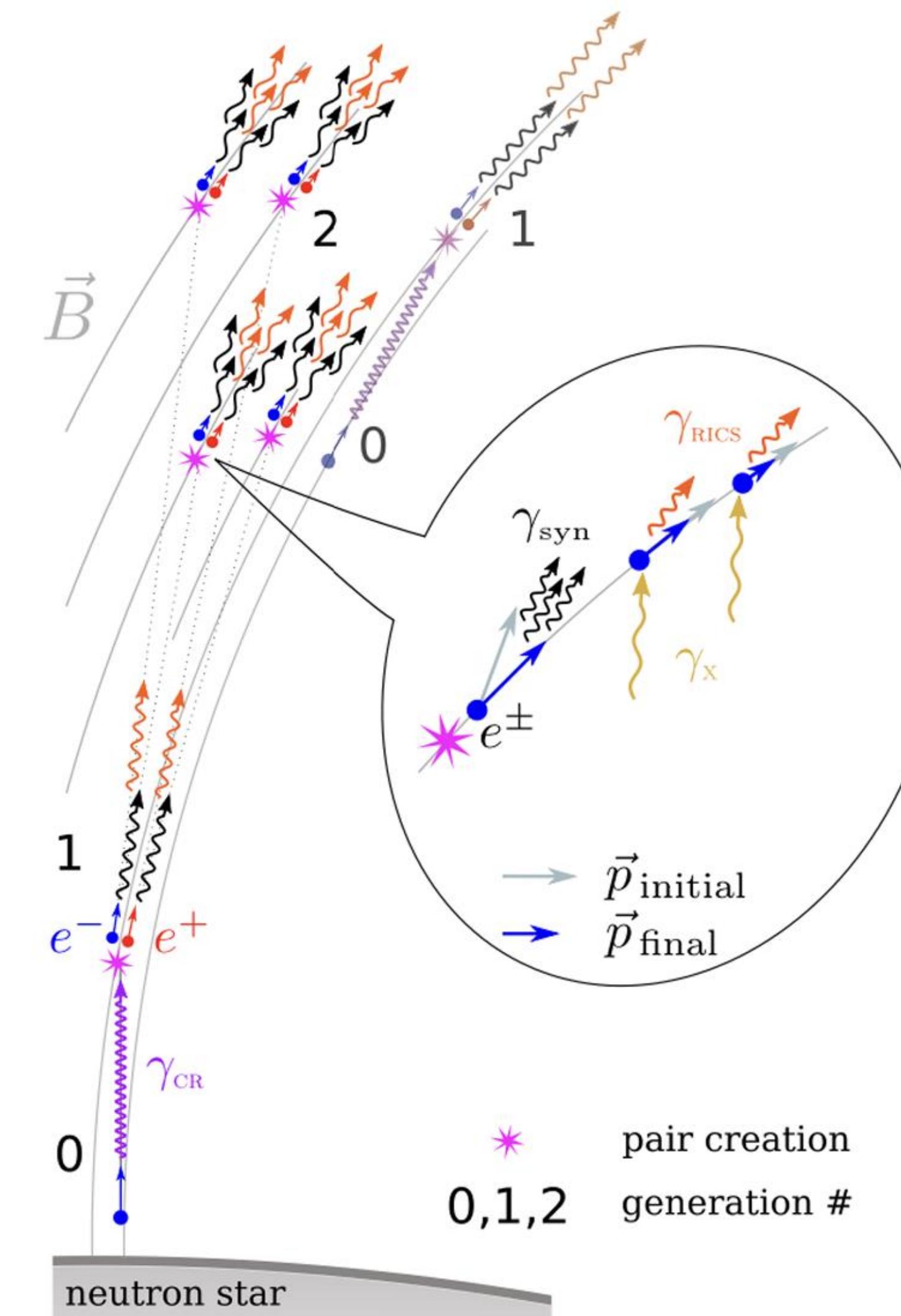
Charge density satisfies:

$$\nabla \cdot \mathbf{E} = 4\pi\rho$$

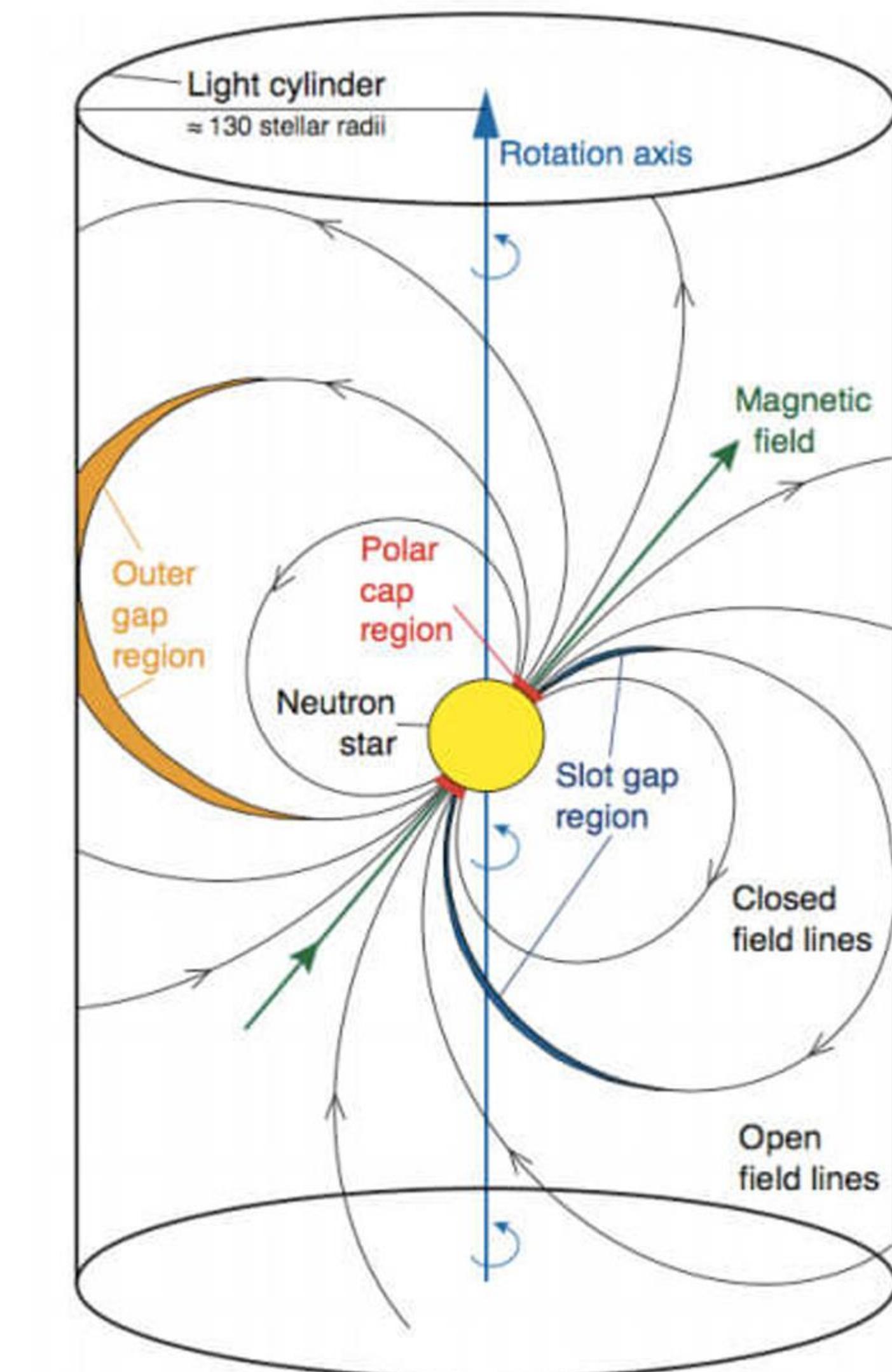
$\rightarrow$

$$\rho_{\text{GJ}} = -\frac{\boldsymbol{\Omega} \cdot \mathbf{B}}{2\pi c} \frac{1}{1 - (\Omega r/c)^2 \sin^2 \theta}$$

$$n_{\text{GJ}} \equiv \rho_{\text{GJ}}/e \approx 7 \times 10^{10} \times \left( \frac{B_z}{10^{12} G} \right) \left( \frac{P}{1s} \right)^{-1} \text{cm}^{-3}$$



Goldreich-Julian density (Goldreich & Julian 1969)



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Goldreich-Julian density (Goldreich & Julian 1969)

Corotation: limited because

$$|\boldsymbol{\Omega} \times \mathbf{r}| < c$$

$\rightarrow$  Light cylinder (LC):

$$R_{\text{LC}} = c/\Omega$$

Magnetic field lines

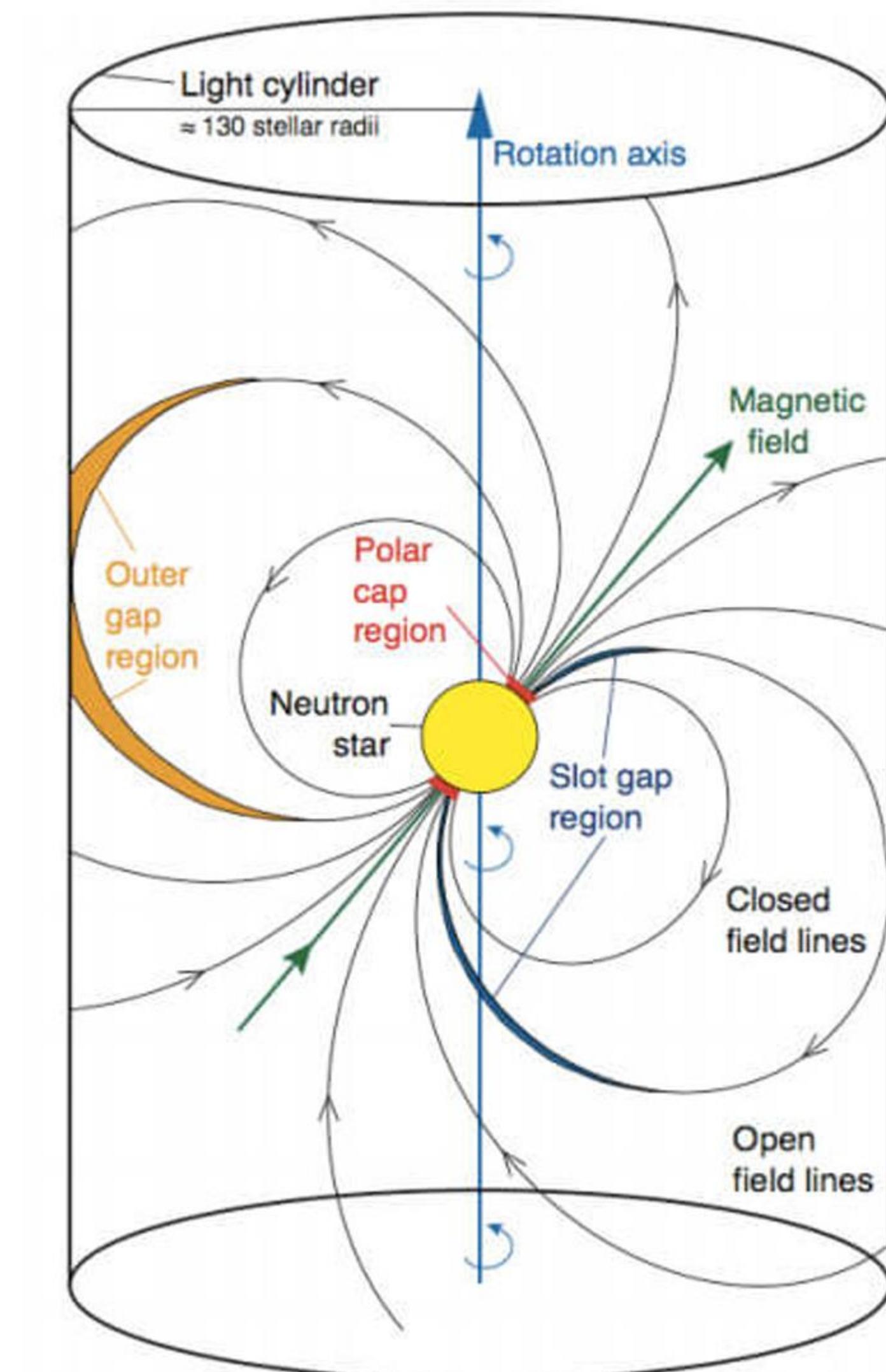
$\triangleright$  Closed within LC:

**Closed field lines**

$\triangleright$  Not closed within LC:

**Open field lines**

Feet of open field lines on pulsar surface: **Polar cap**.



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## (ii) Introduction to models

### (1) From Charge density driven to Current density driven:

From previous pages, we know when  $\rho \neq \rho_{\text{GJ}}$  at somewhere, the magnetosphere is no longer static (non-force-free, non-FFE).

But for open field lines region, the magnetosphere is naturally “non-static”: Open field lines **twist** at light cylinder  $\rightarrow$  always requires magnetospheric currents.

Use current density as indication for acceleration’s happening.

Introduce  $\alpha = j_{\parallel}/(\rho_{\text{GJ}}c)$

$0 < \alpha < 1$ : (mild relativistic  $\rho = \rho_{\text{GJ}}$  flow) or (ultra-relativistic  $\rho < \rho_{\text{GJ}}$  flow)  $\rightarrow$  no lack for charge

$\alpha > 1$ :  $|\rho| > |\rho_{\text{GJ}}|$  flow  $\rightarrow$  charge starvation  $\rightarrow$  parallel electric field arises

$\alpha < 0$ : net charge decrease  $\rightarrow$  charge starvation  $\rightarrow$  parallel electric field arises

## (2) Ruderman-Sutherland (RS) model v.s. Space-Charge-Limited-Flow (SCLF) model:

RS model (Ruderman & Sutherland 1975): no supplement of plasma from pulsar surface.  
(With isolated “sparks” → can explain subpulse drifting)

SCLF mode (Arons & Scharlemann 1979...): ions & electrons supplied by pulsar surface/atmosphere.

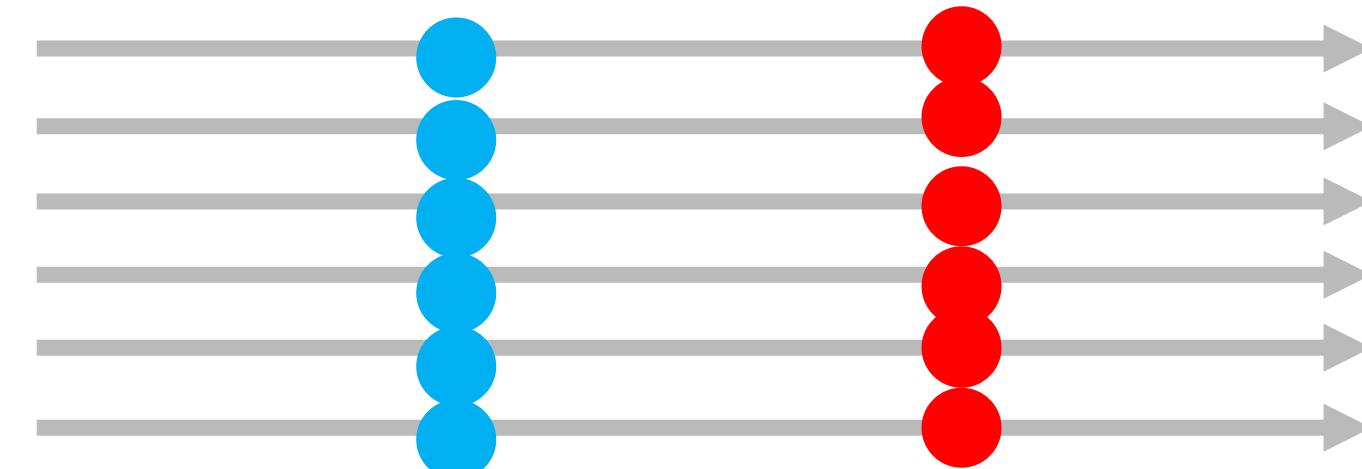
(Different in binding energy at pulsar surface)

## (3) Coherent radio emission mechanism:

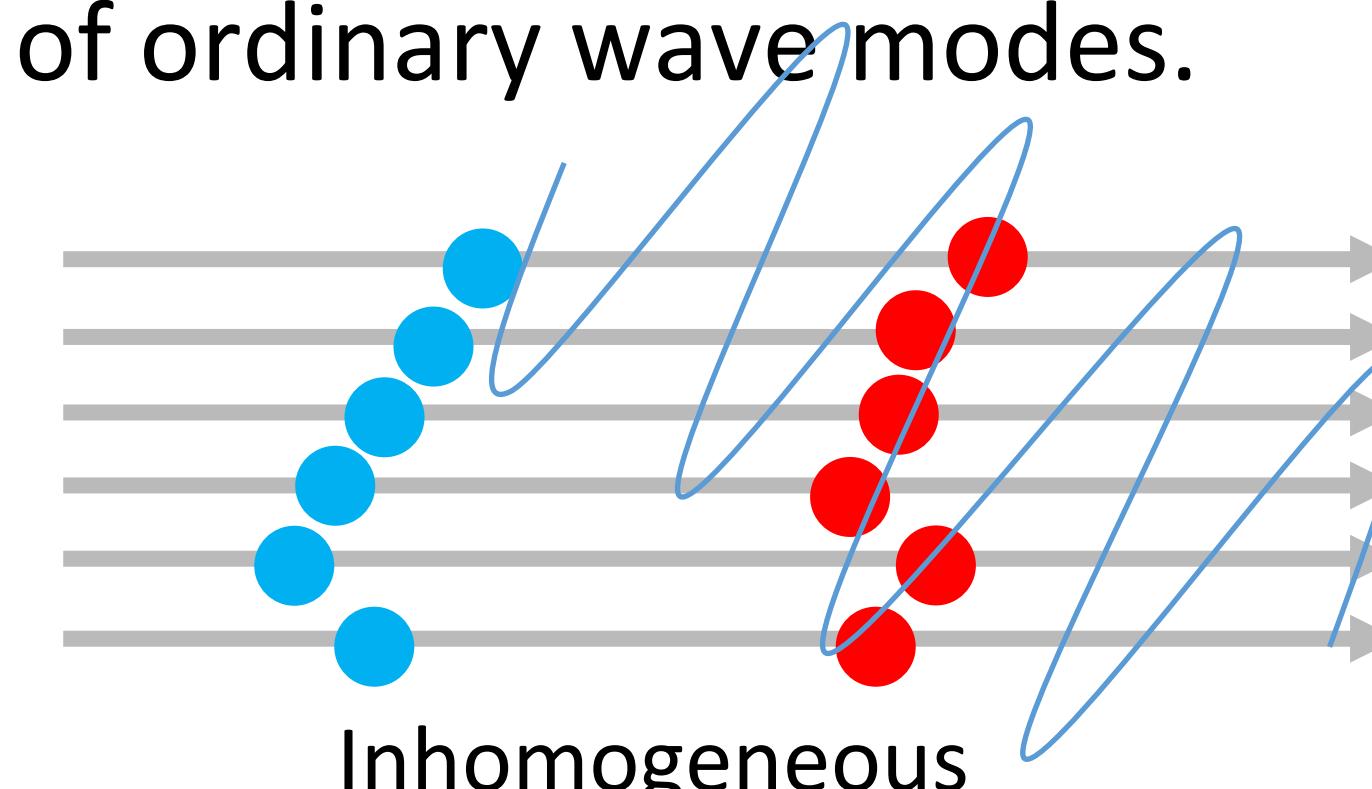
This paper:  
2D & 3D simulations  
On discharge processes

Simulation by Philippov, Timokhin & Spitkovsky 2020:

Spatial inhomogenous discharge causes excitation of ordinary wave modes.



Homogeneous: Oscillation // Magnetic field



## II. Simulation setups

### (1) EM dynamics

Unperturbed (Force-Free, FFE):  $\mathbf{B}_{\text{FFE}} = \mathbf{B}_0 + \mathbf{B}_\varphi$ ,  $\mathbf{E}_{\text{FFE}} = -\boldsymbol{\Omega} \times \mathbf{r} \times \mathbf{B}_0/c$   $\rho_{\text{GJ}} = \nabla \cdot \mathbf{E}_{\text{FFE}}/4\pi$

Corrections:  $\frac{\partial}{\partial t} \delta \mathbf{E} = c \nabla \times \delta \mathbf{B} - 4\pi(\mathbf{j} - \mathbf{j}_{\text{mag}})$ ,  $\nabla \cdot \delta \mathbf{E} = 4\pi(\rho - \rho_{\text{GJ}})$   
 $\frac{\partial}{\partial t} \delta \mathbf{B} = -c \nabla \times \delta \mathbf{E}$ .

Polar cap:  $R_{\text{PC}} = R_\star \sqrt{R_\star/R_{\text{LC}}}$   $R_{\text{LC}} = cP/2\pi$

Two stationary solutions:

- (i)  $\delta \mathbf{E} = \delta \mathbf{B} = 0$ , fully force free,  $\mathbf{j} = \mathbf{j}_{\text{mag}}$
- (ii)  $\mathbf{j} = 0$ ,  $\delta \mathbf{B} = -B_\varphi$ , no magnetic field twist

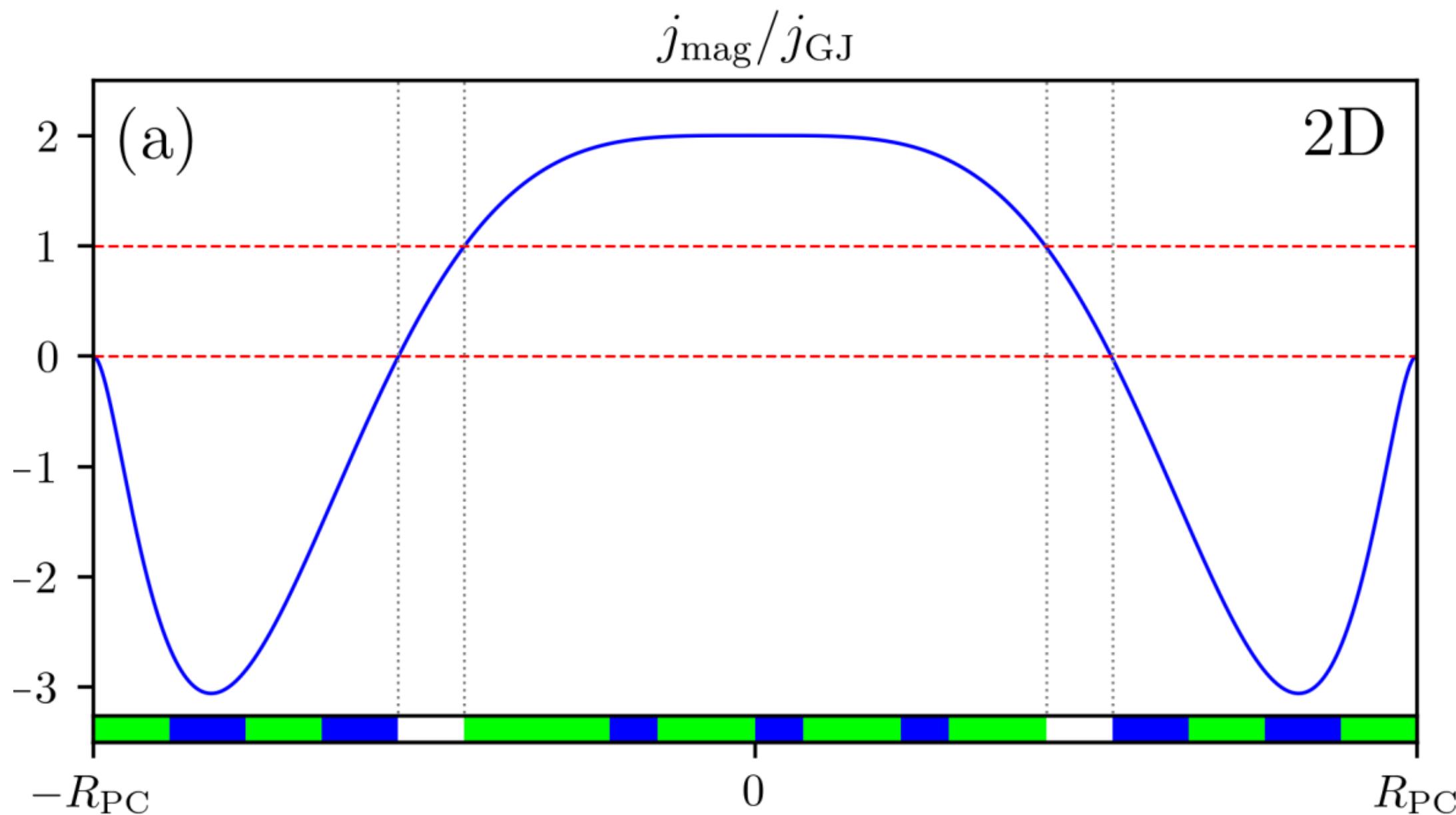
← Abundant plasma everywhere  
← No plasma loading

## (2) Magnetospheric current distribution: follow Gralla et al. 2016, 2017

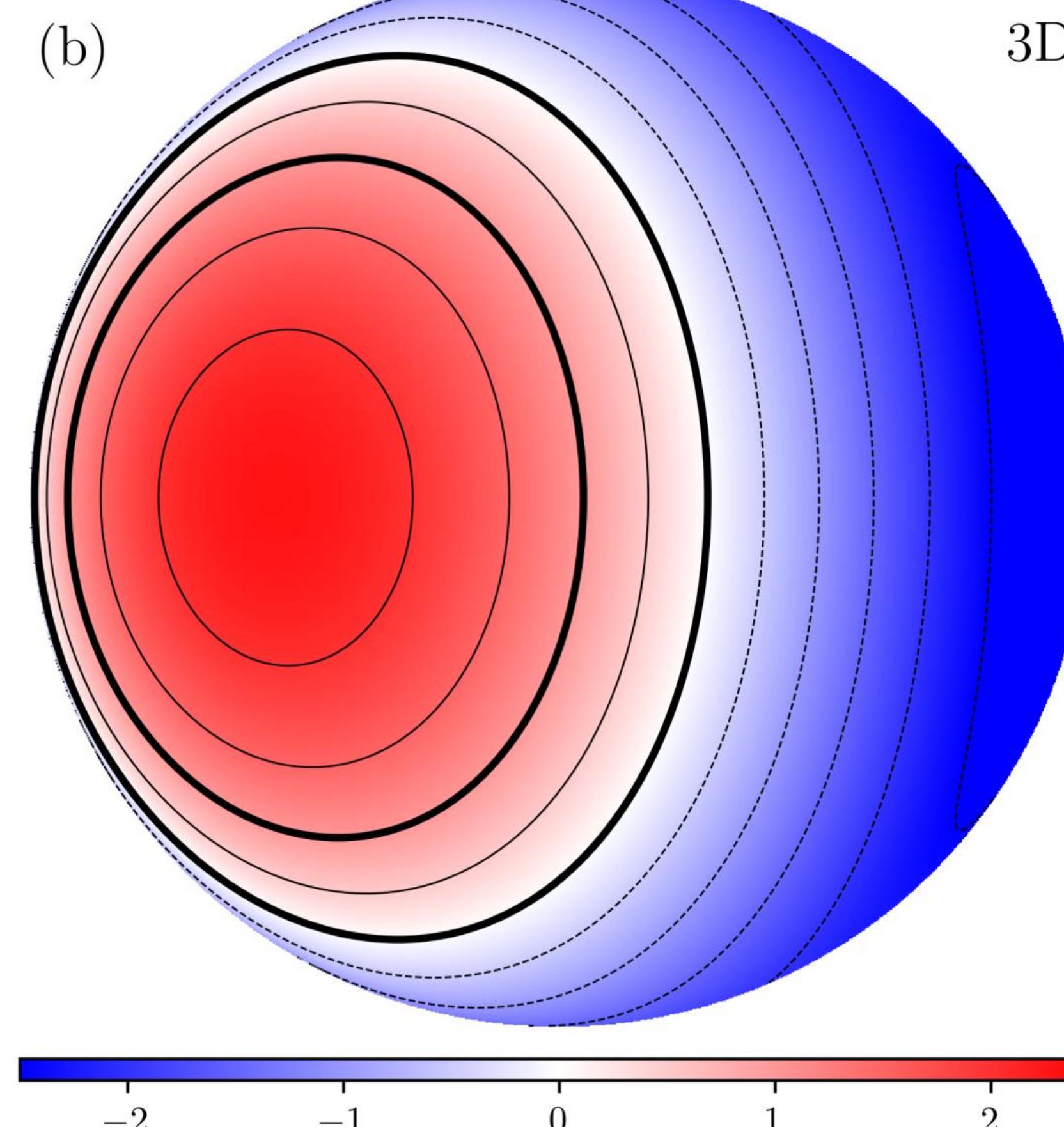
2D:  $j_{\text{mag}}^{2\text{D}}(x) = 2 - C_1 x^4 + C_2 x^6 + C_3 x^{40}$

$$\int_0^{R_{\text{PC}}} j_{\text{mag}}^{2\text{D}} dx = 0, \quad j_{\text{mag}}^{2\text{D}}(1) = 0, \quad \frac{d}{dx} j_{\text{mag}}^{2\text{D}} \Big|_{x=1} = 0$$

$$x = r_{\perp}/R_{\text{PC}}$$



3D:  $\frac{j_{\text{mag}}^{3\text{D}}}{\rho_{\text{GJ}} c}(\theta, \phi) \approx \frac{1}{(1 - \Omega_Z/\Omega)} [J_0(\arcsin(\theta/\sqrt{\alpha_0})) - J_1(\arcsin(\theta/\sqrt{\alpha_0})) \tan i \cos \phi].$



(3) QED pair creation  $\rightarrow$  leads to large multiplicity  $\mathcal{M} = n_{\pm}/n_{\text{GJ}} \gg 1$

Emission in polar cap: synchrotron curvature radiation.

$$\frac{dN_{\text{ph}}}{dtd\varepsilon} = \frac{1}{\sqrt{3}\pi} \frac{e^2}{\hbar^2 c} \frac{1}{\gamma_b^2} \int_{\frac{\varepsilon^*_{\text{ph}}}{\varepsilon^*_{\text{ph}}}}^{\infty} K_{5/3}(x) dx,$$

$$\varepsilon_{\text{ph}}^* = \frac{3}{2} \frac{\hbar c}{\rho_c} \gamma_b^3.$$

Cross section for pair creation:

$$\frac{d\sigma}{dz} = 0.23 \frac{B}{B_q} \sin \psi \frac{\alpha_F}{\lambda_c} \exp\left(-\frac{8}{3\chi}\right) \Theta(\tilde{\varepsilon}_{\text{ph}} \sin \psi - 2),$$

$$\begin{aligned} \chi &= (B/B_q) \tilde{\varepsilon}_{\text{ph}} \sin \psi & \tilde{\varepsilon}_{\text{ph}} &= \varepsilon_{\text{ph}}/m_e c^2 \\ B_q &= m_e^2 c^3 / e \hbar \approx 4.41 \times 10^{13} \text{G} \end{aligned}$$

Secondary particles' velocity:

$$u_{||} = \frac{|\cos \psi_a| (\tilde{\varepsilon}_{\text{ph}}^2 - 4)^{1/2}}{\left(\tilde{\varepsilon}_{\text{ph}}^2 \sin^2 \psi_a + 4 \cos^2 \psi_a\right)^{1/2}} \sim \frac{1}{\sin \psi_a} \sim 10^2 - 10^3,$$

Pair creation & emission energy scales described in 3 gamma parameters:

$$\gamma_{\text{PC}} = 0.5 (R_{\text{PC}}/d_e^{\text{GJ}})^2$$

$$eE_{\text{PC}} = (2/3) e^2 \gamma_{\text{rad}}^4 / \rho_c^2$$

$$(3/2) \hbar (c/\rho_c) \gamma_{\text{emit}}^3 = m_e c^2$$

$$d_e^{\text{GJ}} = c / \sqrt{4\pi |\rho_{\text{GJ}} e| / m_e}$$

$$\tilde{\varepsilon}_{\text{ph}} = (\gamma / \gamma_{\text{emit}})^3$$

Generally,  $\gamma_{\text{PC}} \gg \gamma_{\text{rad}} \gg \gamma_{\text{emit}}$ .

## (4) Atmosphere

SCLF model: thin electron-ion atmosphere  $\rightarrow$  reservoir of charged particles.

$\approx$  A hot plasma layer at simulation boundary.  $n = n_{\text{peak}} \exp(-z/h)$

(This  $T$  is about  $2.5 \times 10^4$  K.)

$$h = kT/(m_e g) \approx 10d_e^{\text{GJ}}$$

$$n_{\text{peak}} \approx 10n_{\text{GJ}}$$

RS model: no atmosphere.

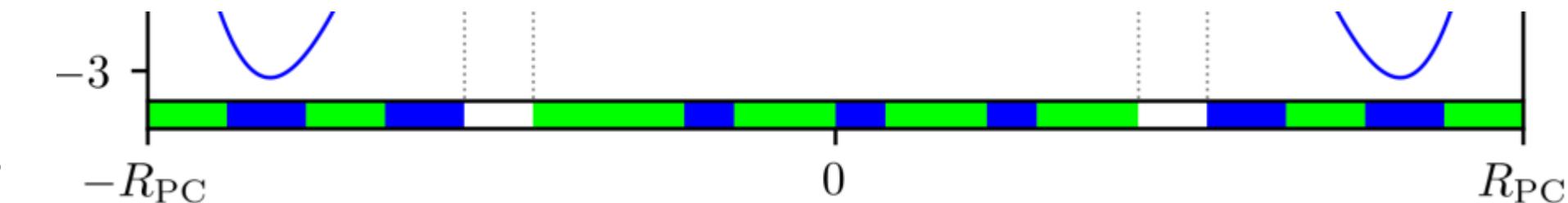
## (5) Initial plasma state

Multiplicity  $\sim$  a few.

$$\mathbf{j} = j_{\text{mag}} \quad \delta \mathbf{E} = 0 \quad \rho_{\text{GJ}} = \rho_{\text{GJ}}^0 (1 + 0.8z/L_z)$$

Initial inhomogeneity: divide polar cap into different patches.

stop injecting initial plasma at different times on neighboring patches.



## (6) Numerical details

Tristan-v2: multi-species radiative PIC code (Hakobyan et al. 2024).

Initial magnetic field: uniform. Curvature of field lines: prescribed. Multiplicity  $< 50$ ...

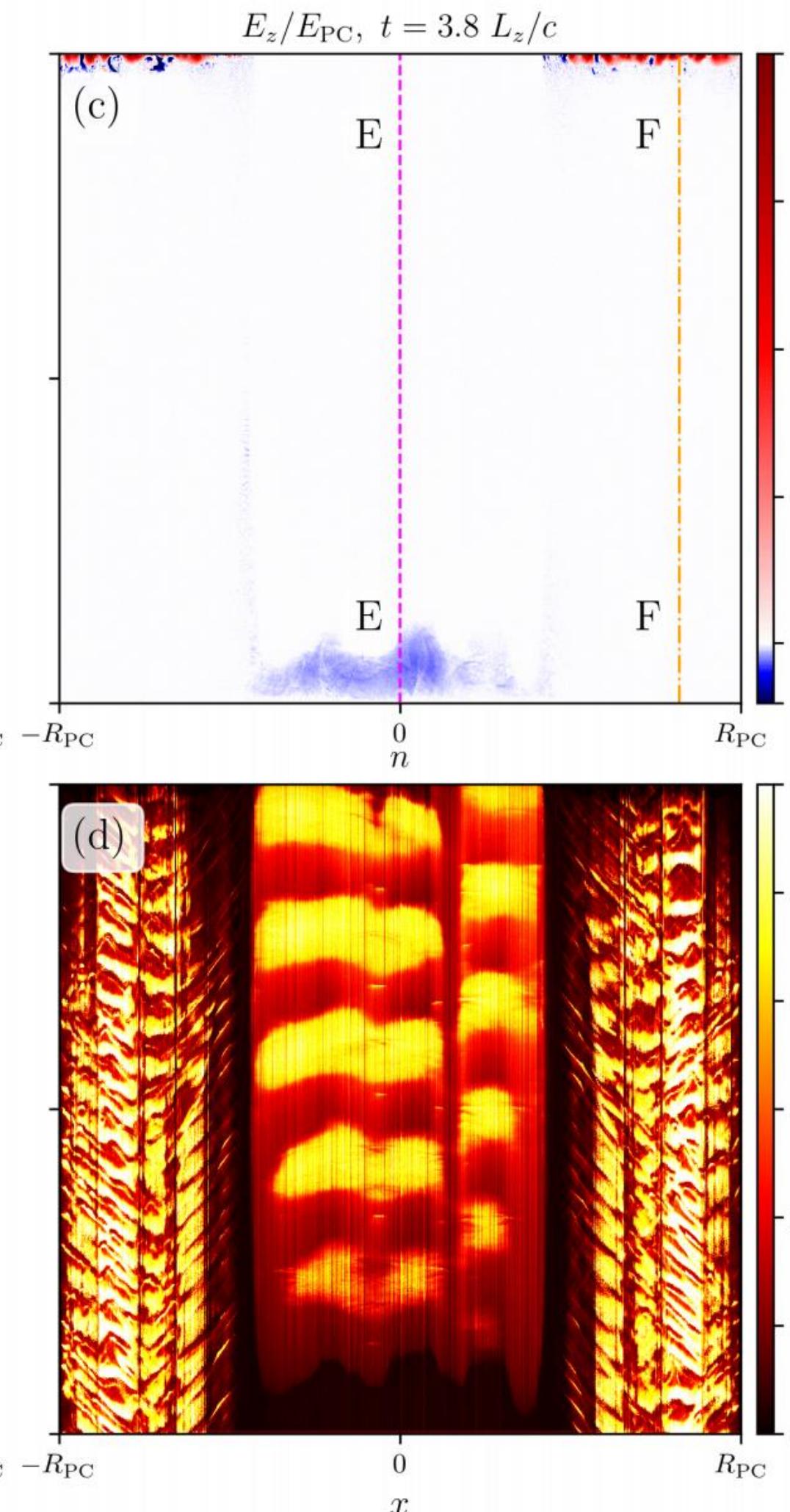
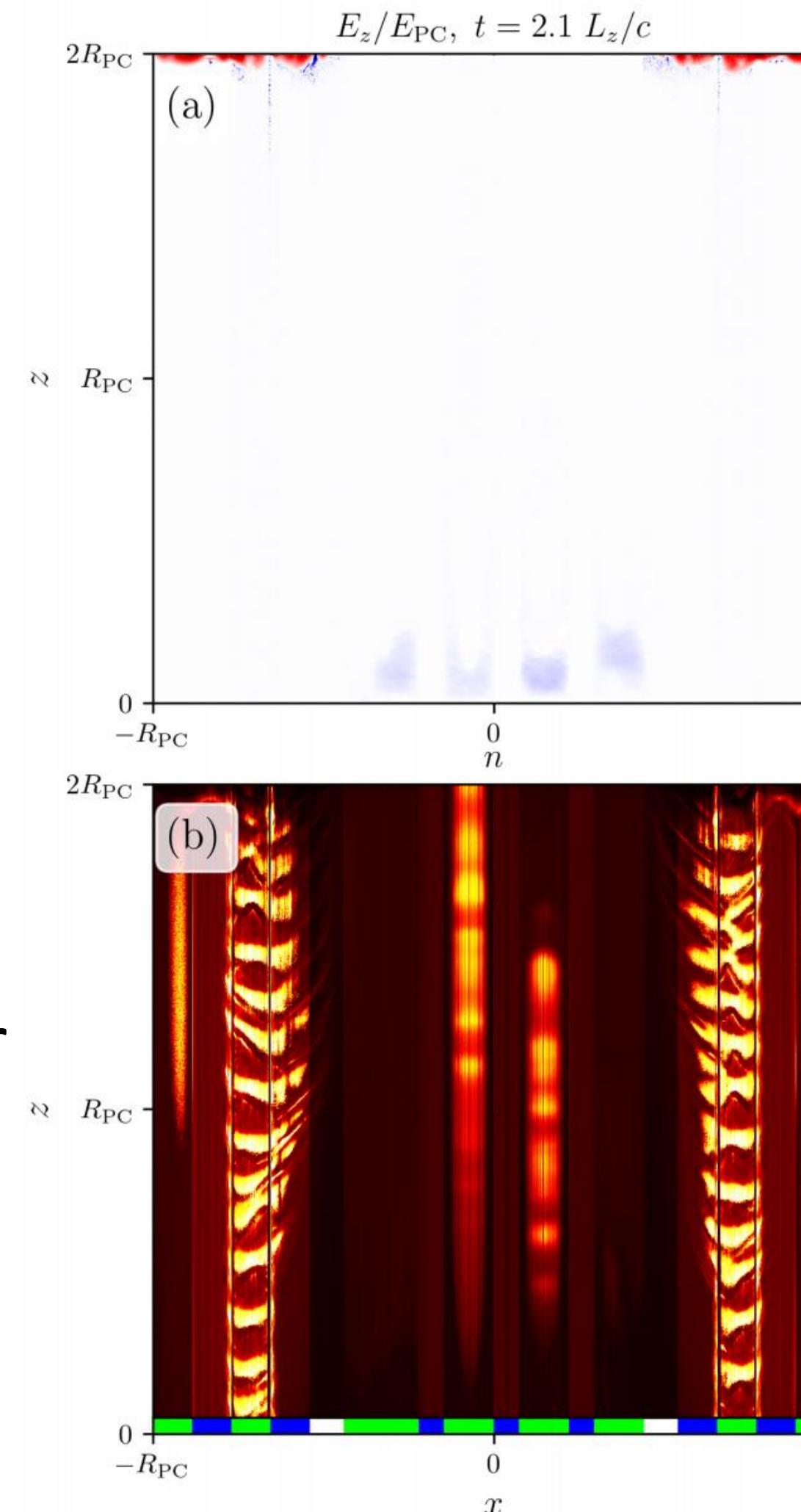
# III. Results

## (1) SCLF

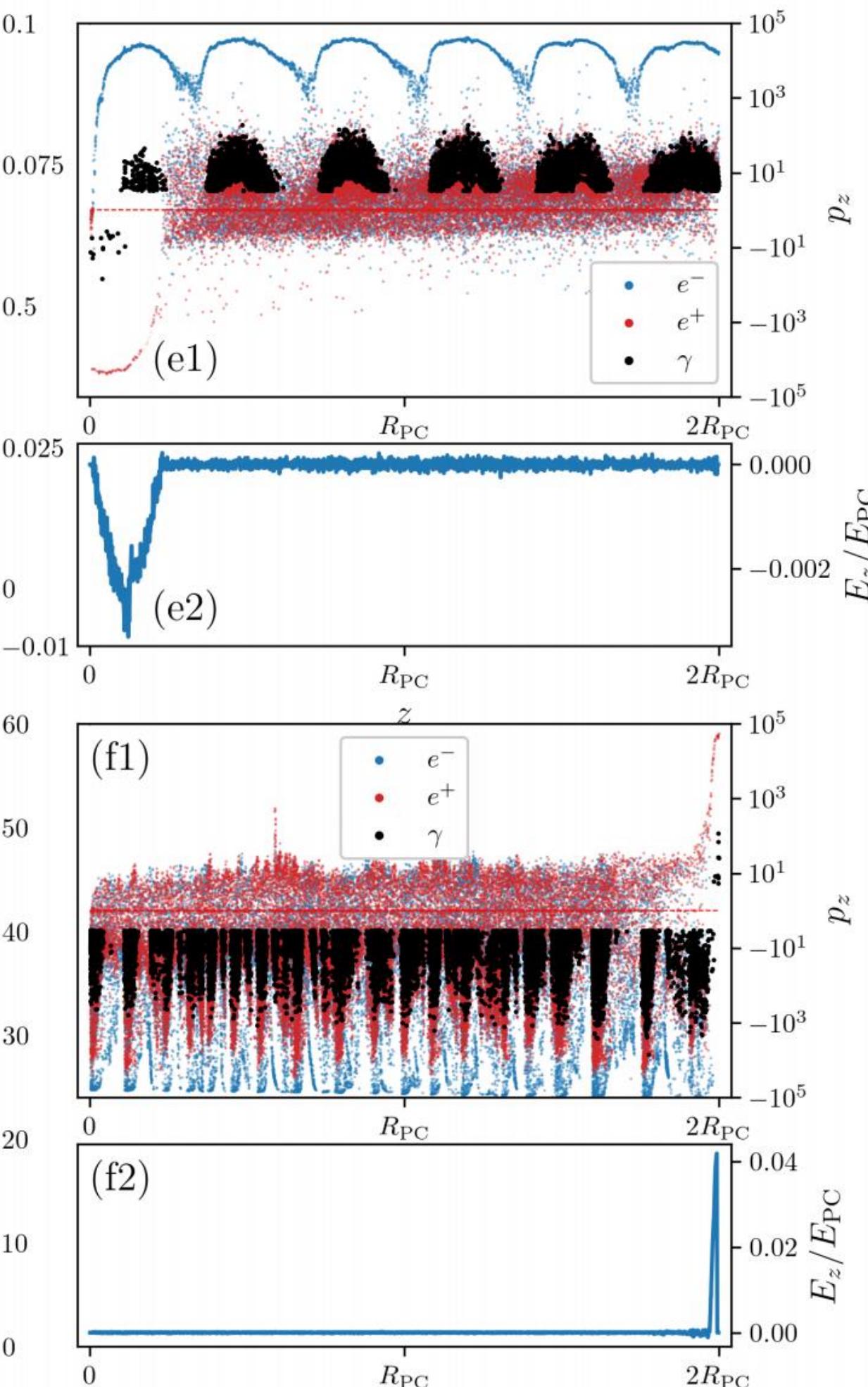
### (1.1) Small gap & Constant field lines' curvature

Dipolar field with multipolar components?

$E_{||}$  field:



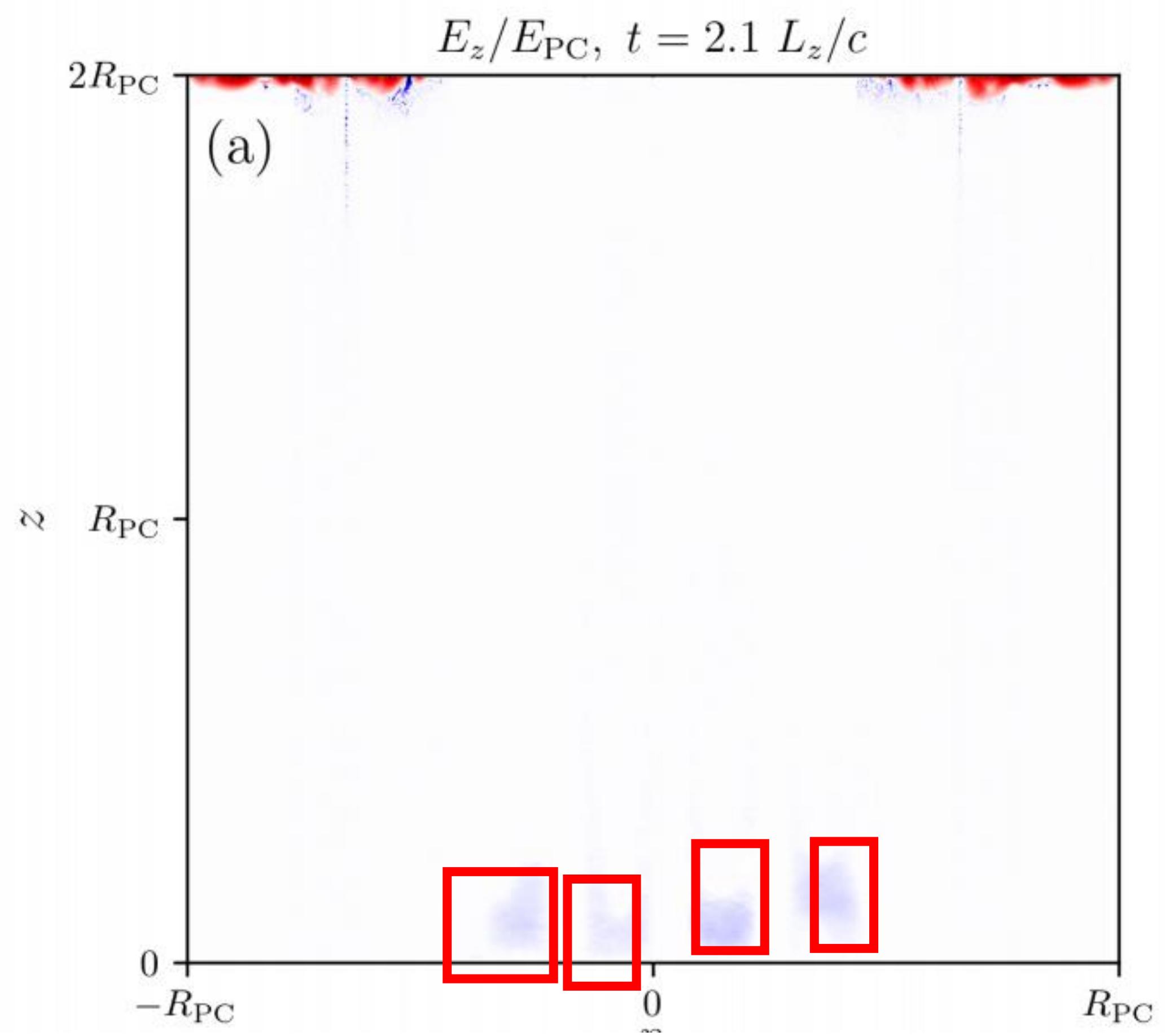
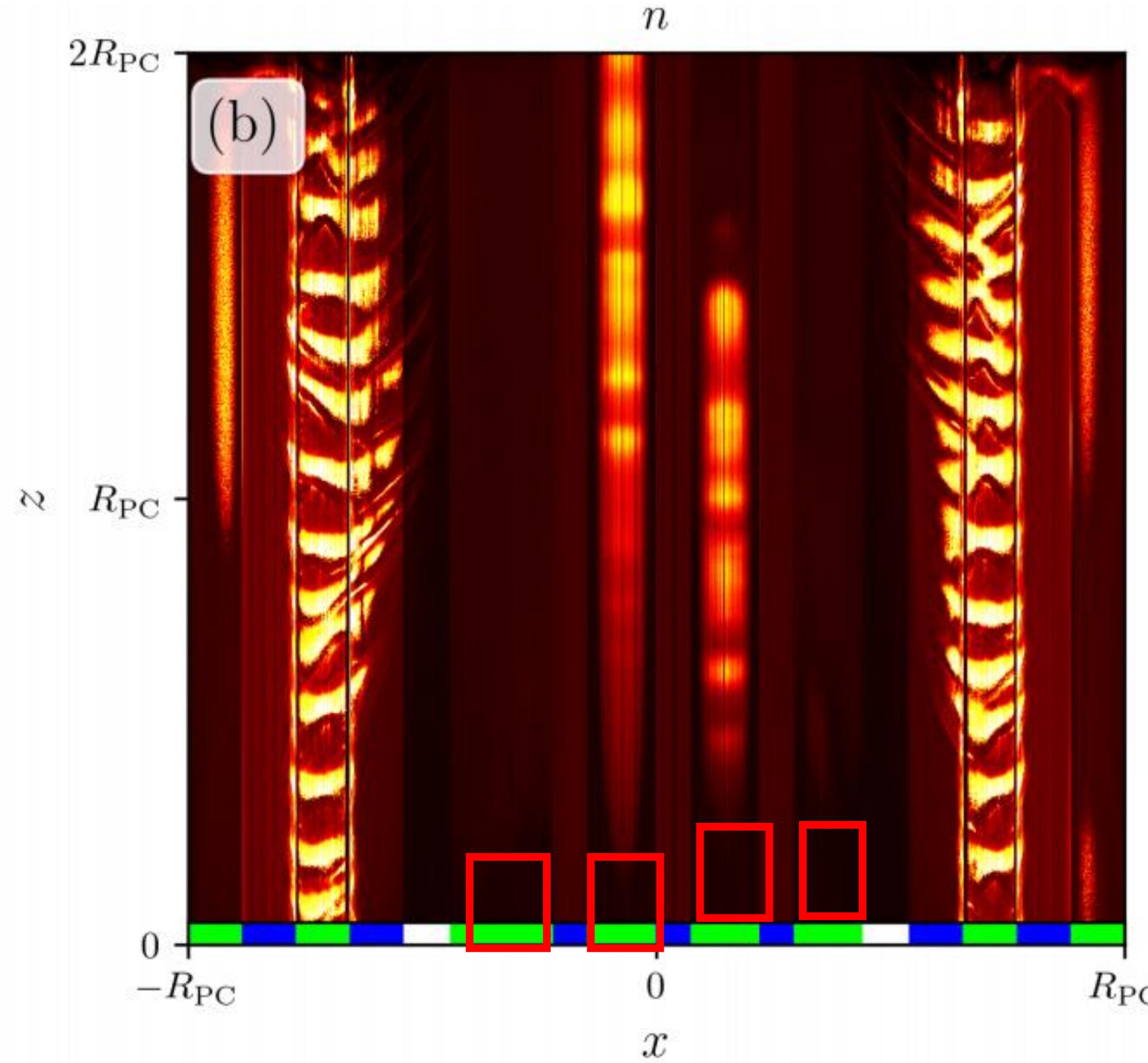
Particle number density:



## Super-GJ region ( $j/j_{\text{GJ}} > 1$ ): gap close to surface.

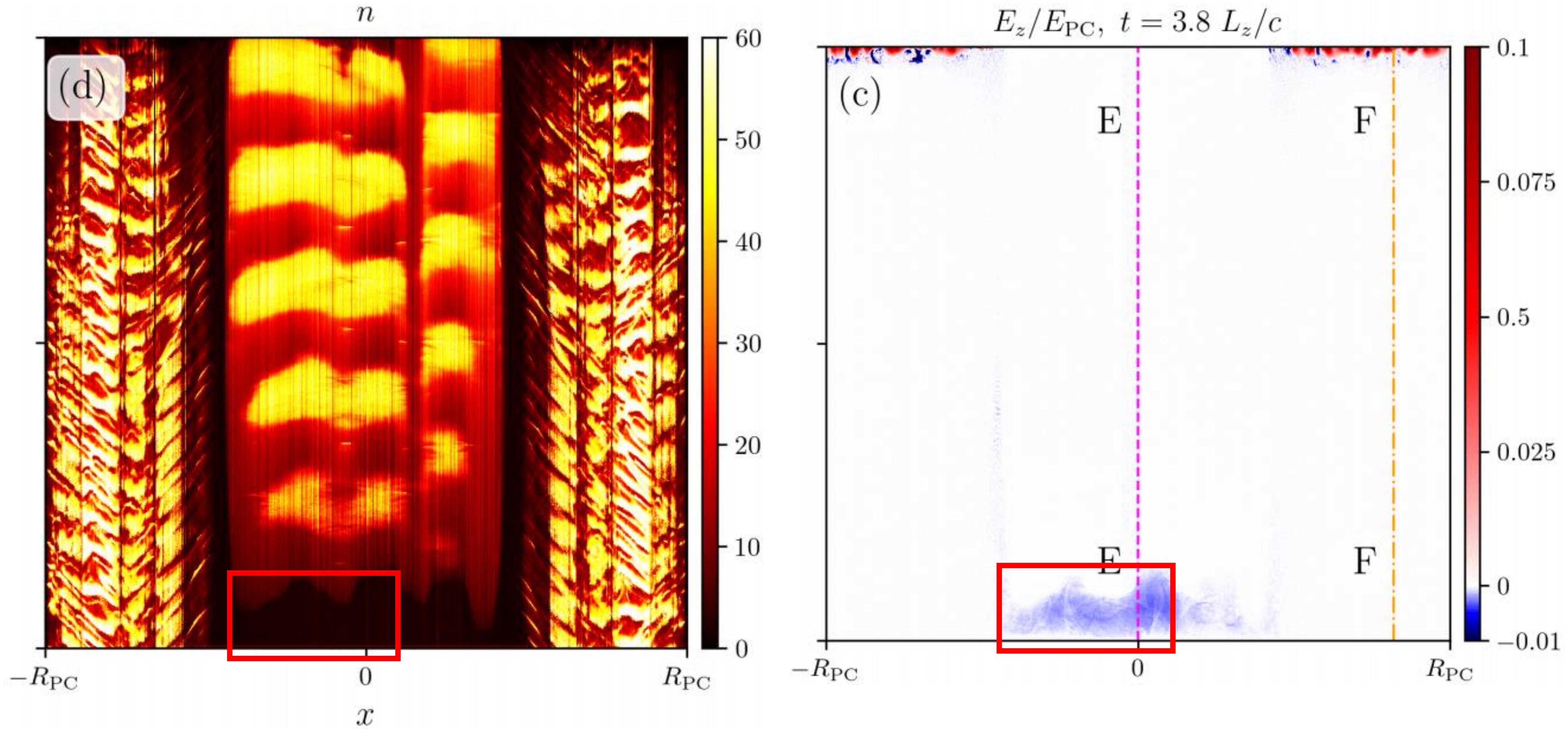
Early time: due to initial inhomogeneity sets, some patches have cleared plasma region, while some have not.

→→ Gaps are also in patches. And they are quasi-stationary.



Super-GJ region ( $j/j_{\text{GJ}} > 1$ ): gap close to surface.

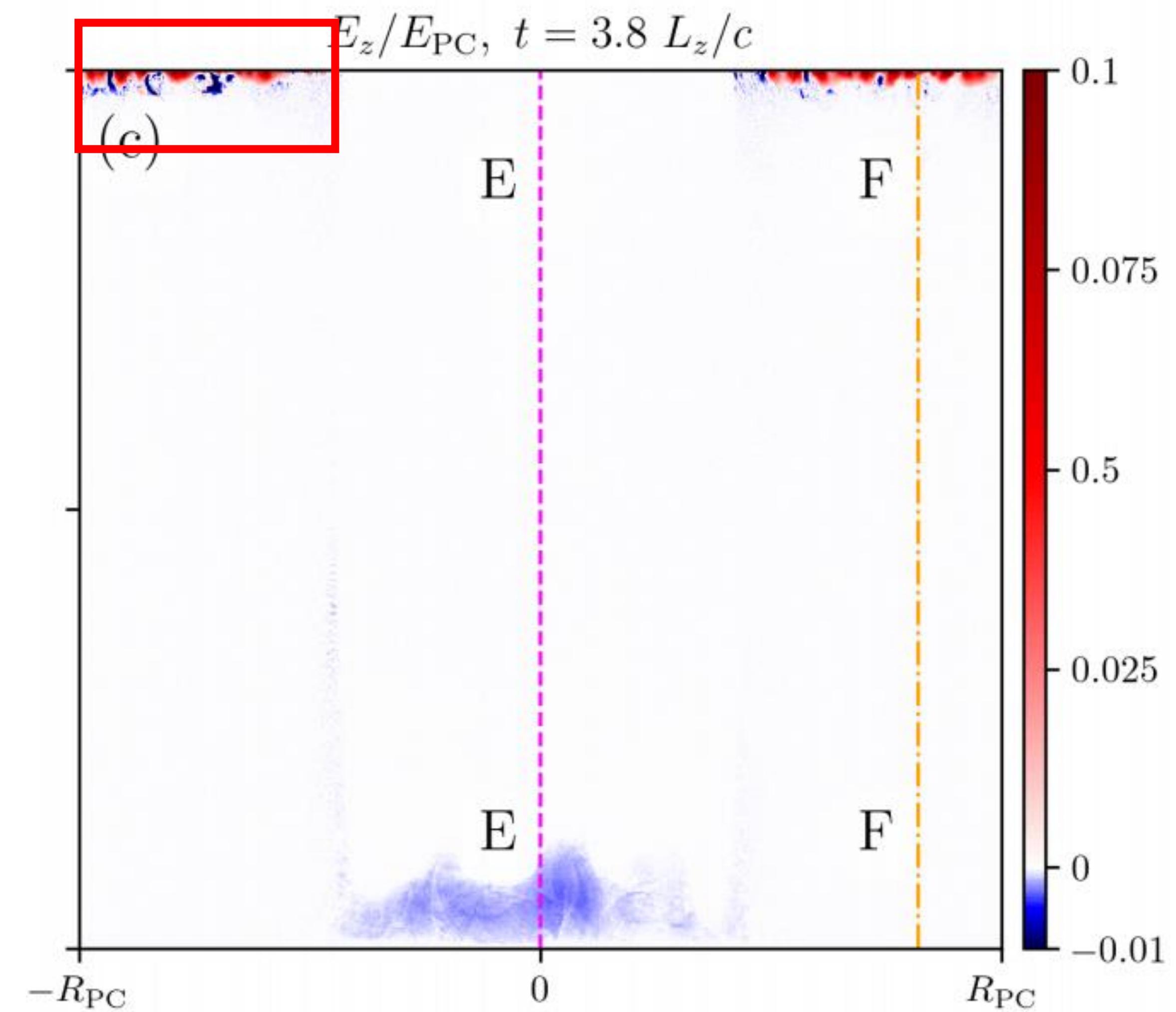
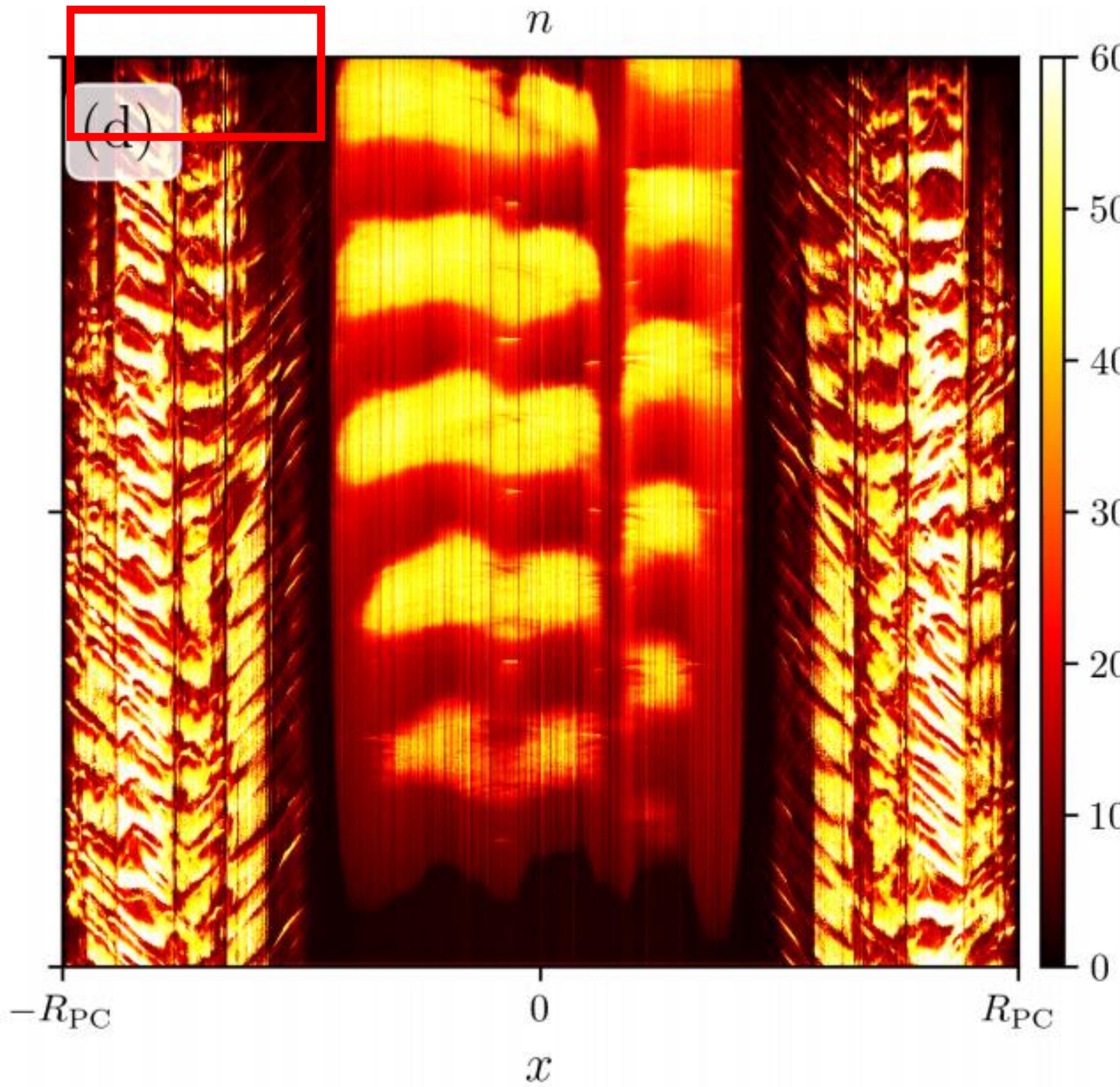
Late time: more patches clear plasma  $\rightarrow$  gaps are connected to **larger pieces**.



Cyclic screening happens  $\rightarrow$  Discharges are intermittent.

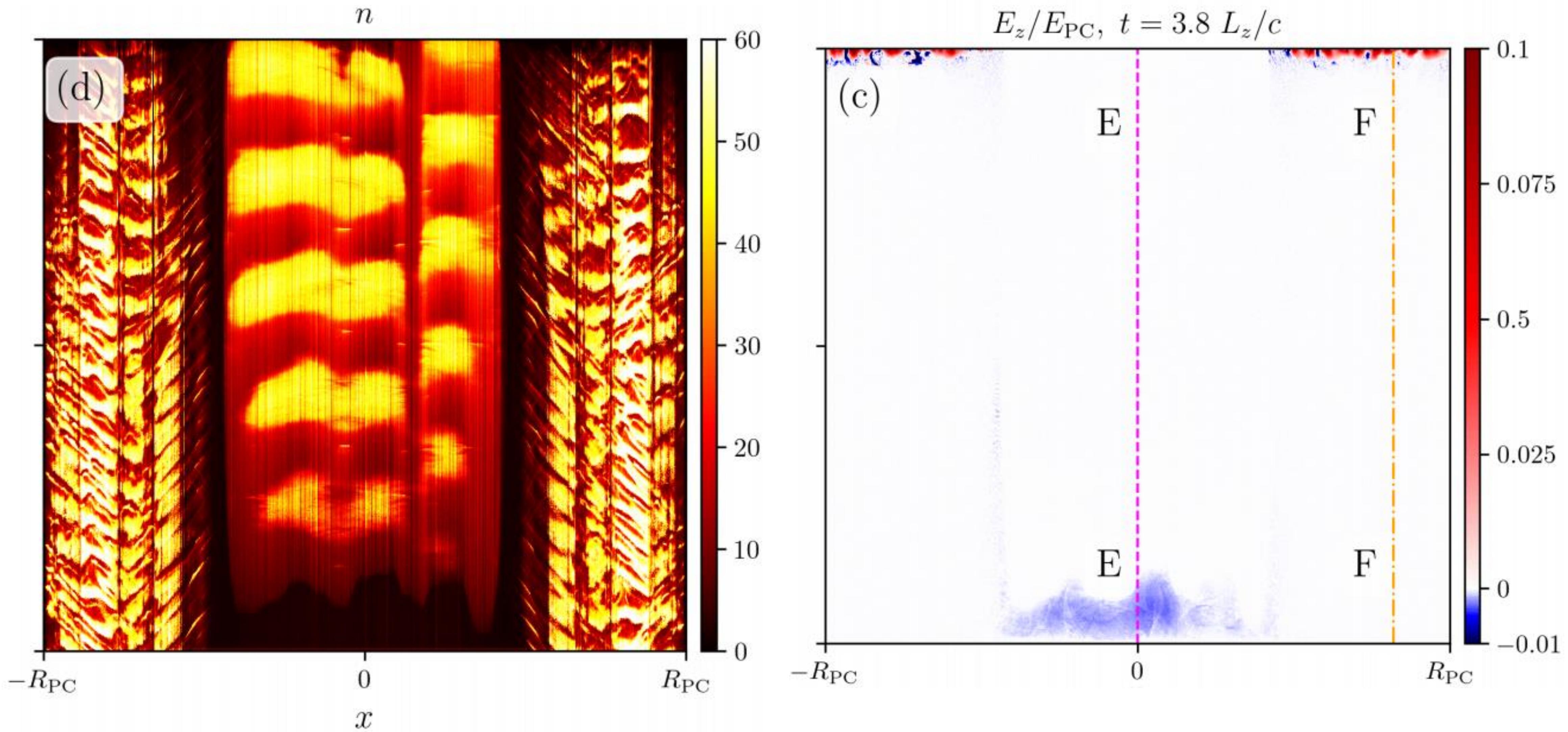
Return current region ( $j/j_{\text{GJ}} < 0$ ): higher gap.

Larger difference in motions of positrons & electrons  $\rightarrow$  stronger electric field  $\rightarrow$  smaller gaps

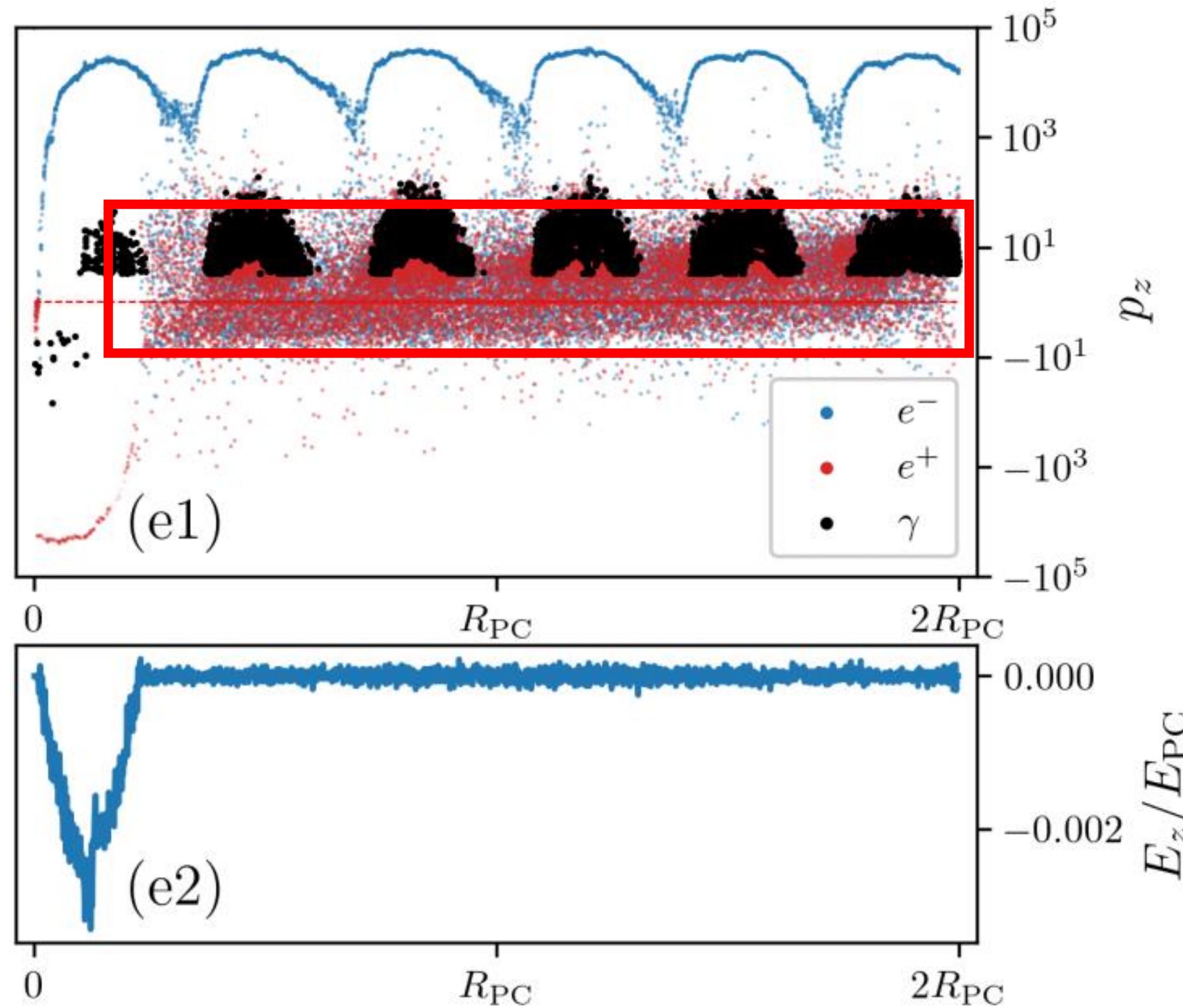


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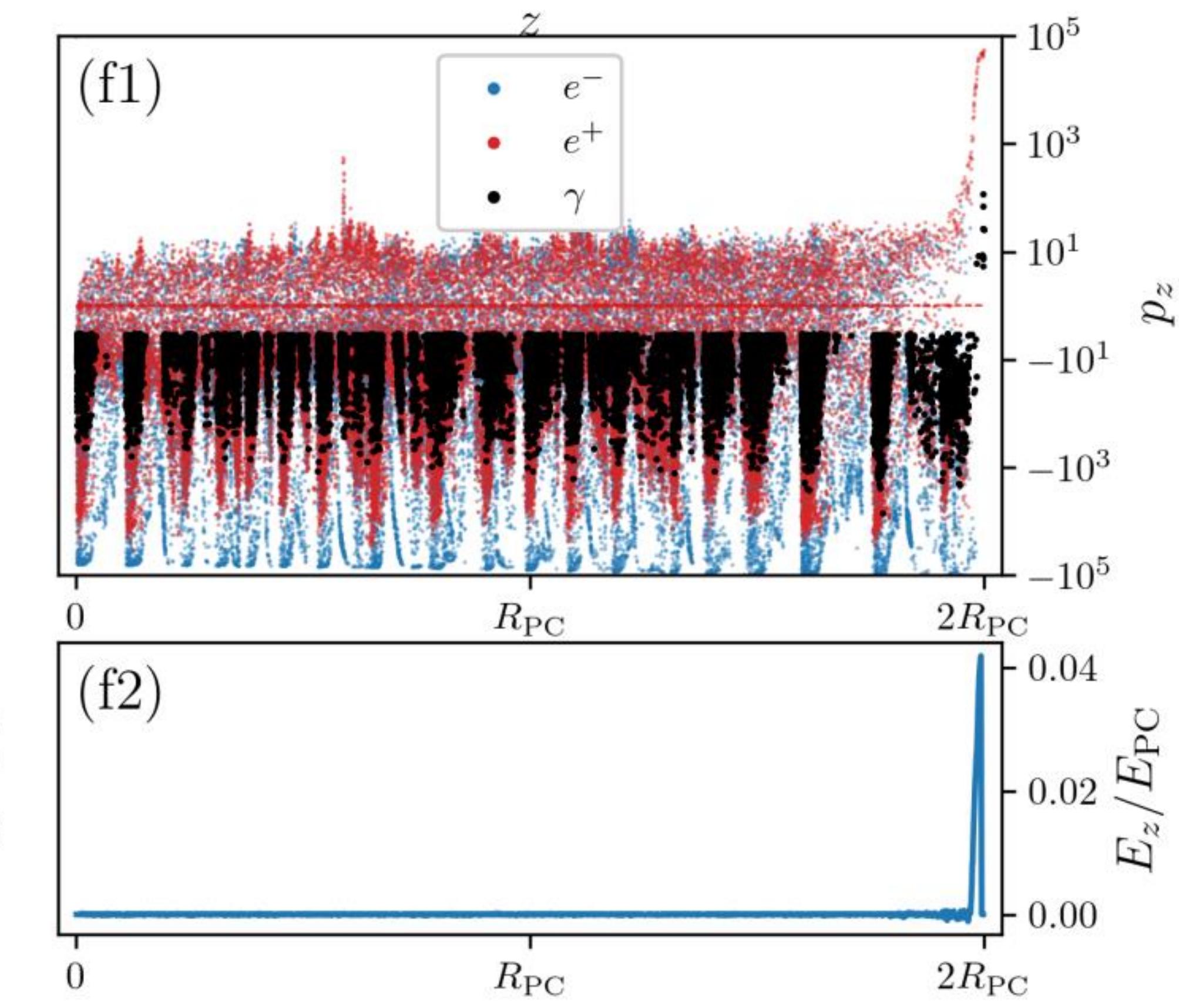
Transverse coherence scale of gaps  $< 2 \cdot l_{\text{gap}}$   $\rightarrow$  Desynchronization of discharges.



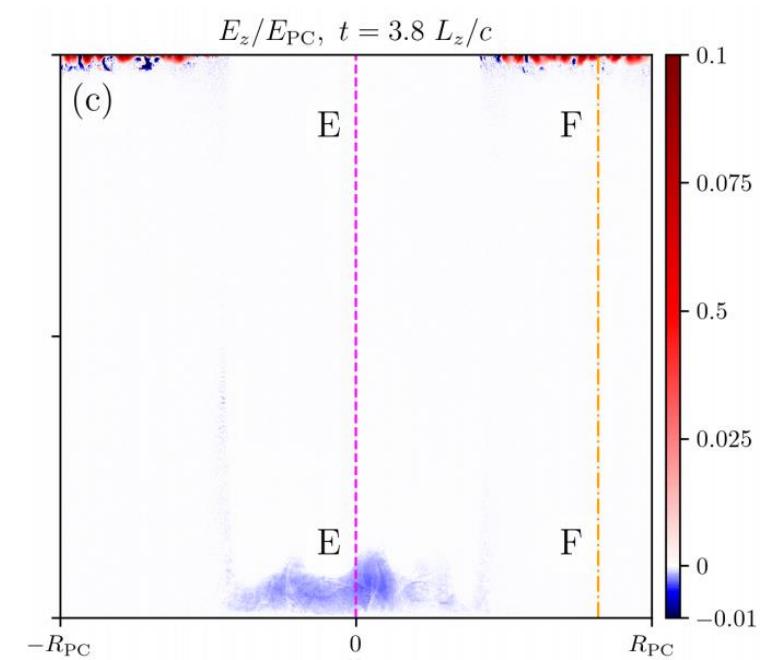
Difference in particle momentums and electric field for two kinds of gaps:



$j/j_{\perp} GJ > 1$



$j/j_{\perp} GJ < 0$



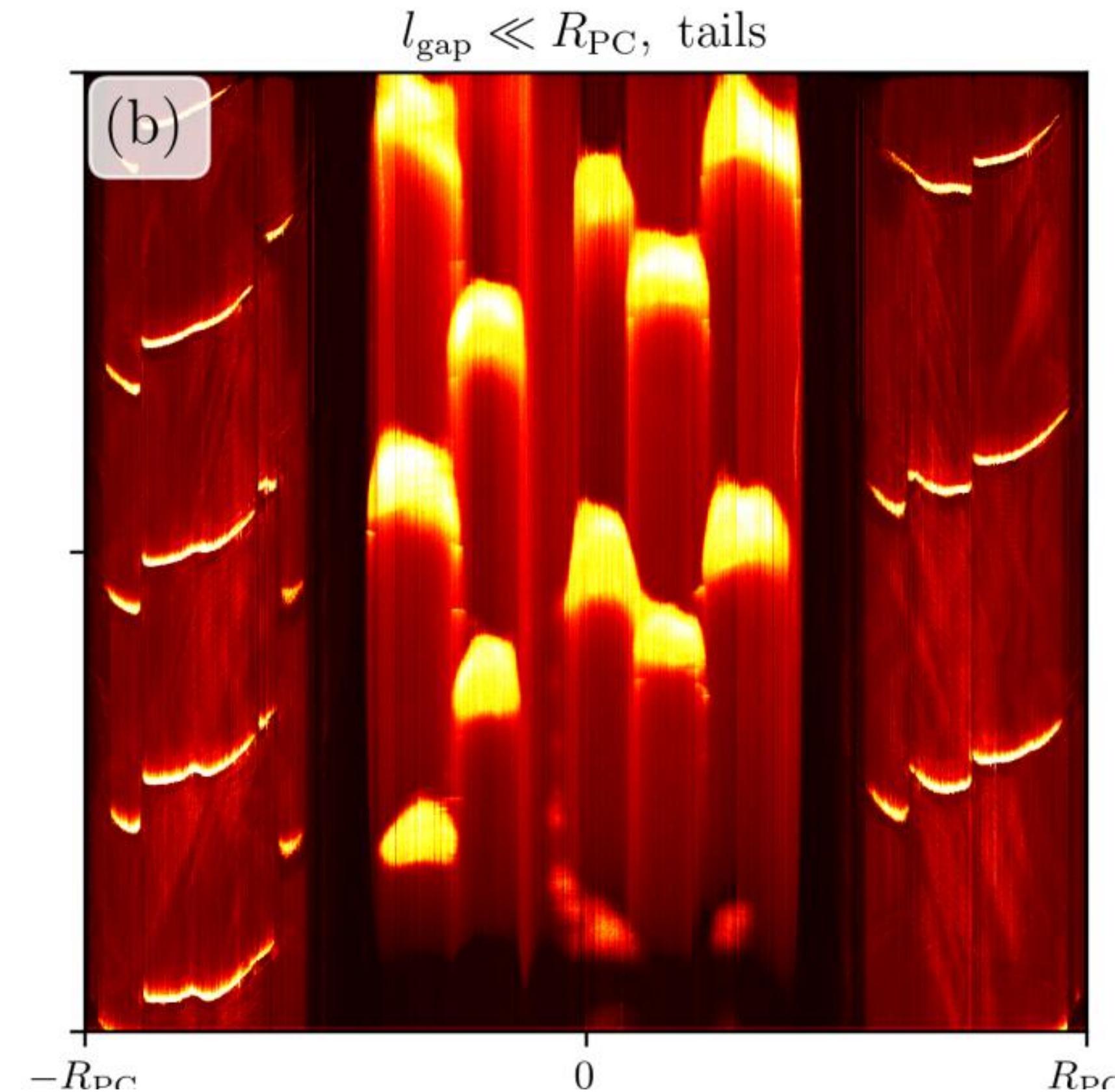
## (1.2) Small gap & Strong Desynchronization (more plasma)

Discharge cyclic period too short → reverse bombardment too strong → surface too hot  
→ can't fit X-ray observation

Actually caused by too low plasma density in simulation.

→ To fix it, the authors inject **additional** extended tails behind escaping clouds of secondary plasma

Particle number  
density:



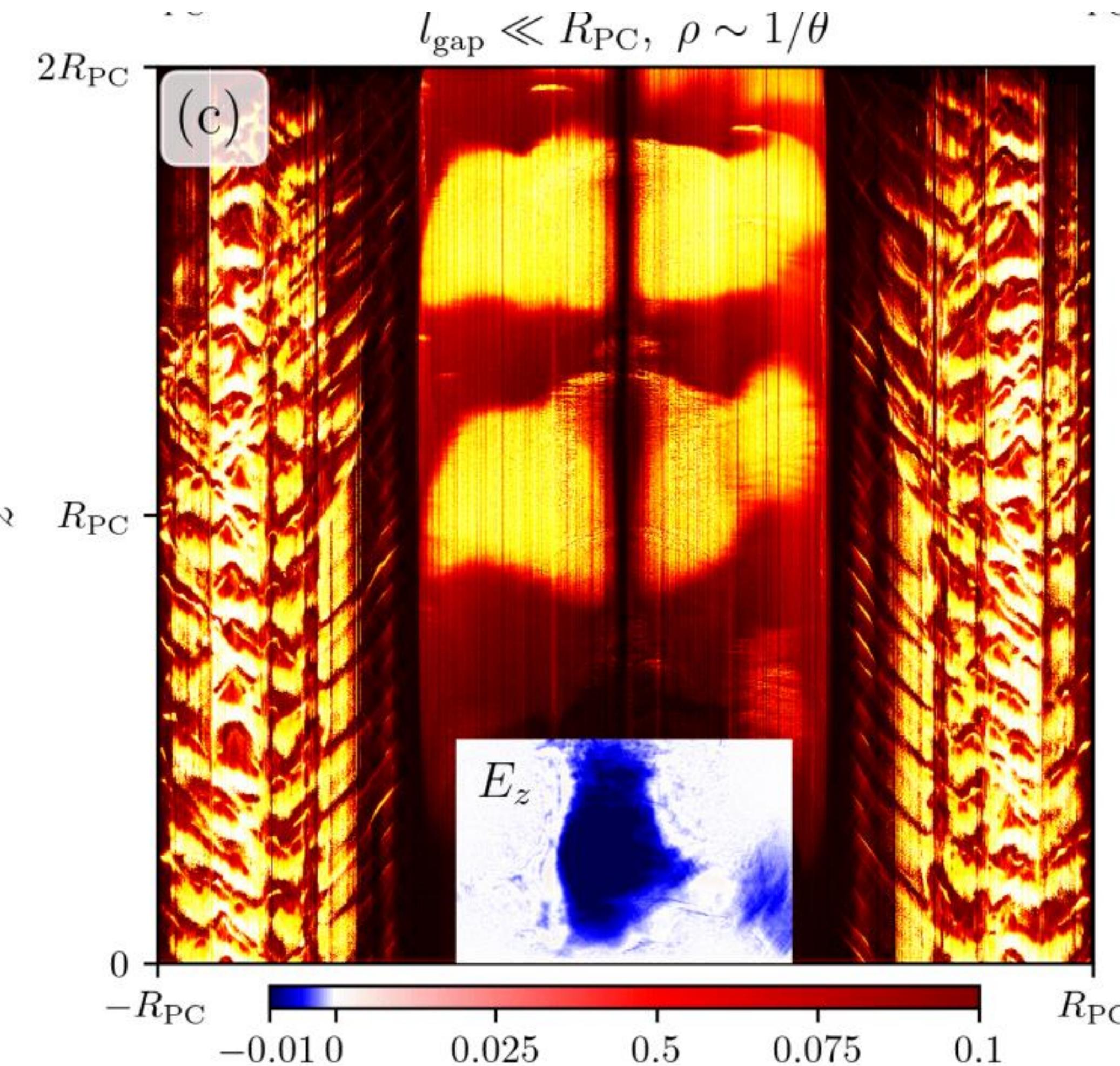
Longer cyclic period  
& Stronger desynchronization

### (1.3) Small gap & Quasi dipolar field

$$\rho_c = \rho_{c,0}(R_{PC}/x)$$

When  $x \rightarrow 0, \rho \rightarrow \infty \rightarrow$  discharge is absent at the center.

Particle number density:



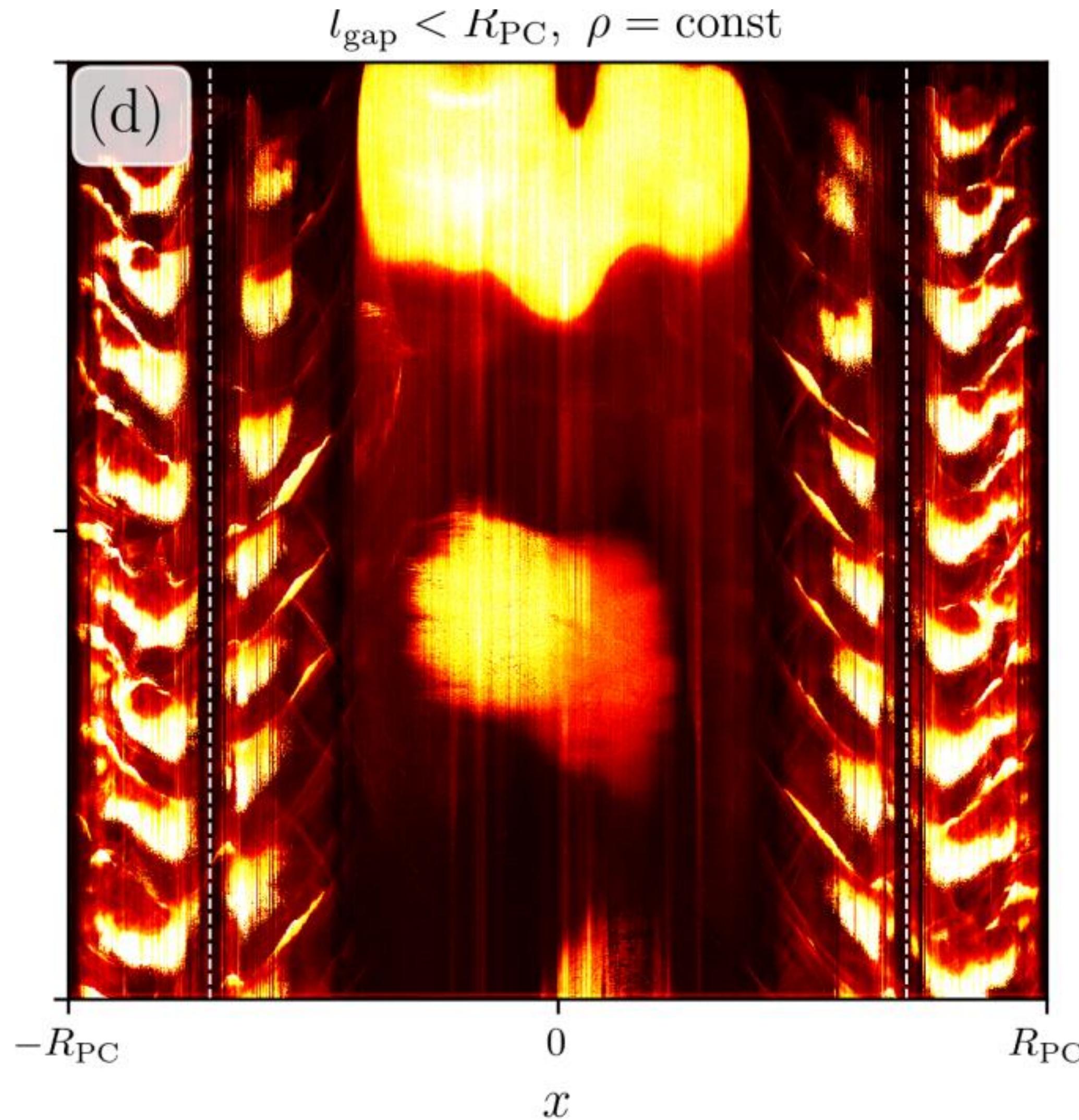
Larger gap.

Front of screening inclined  
→ benefit emission  
(See page 7)

## (1.4) Large gap $\leftarrow$ less energetic pulsars

Smaller electric field for acceleration.

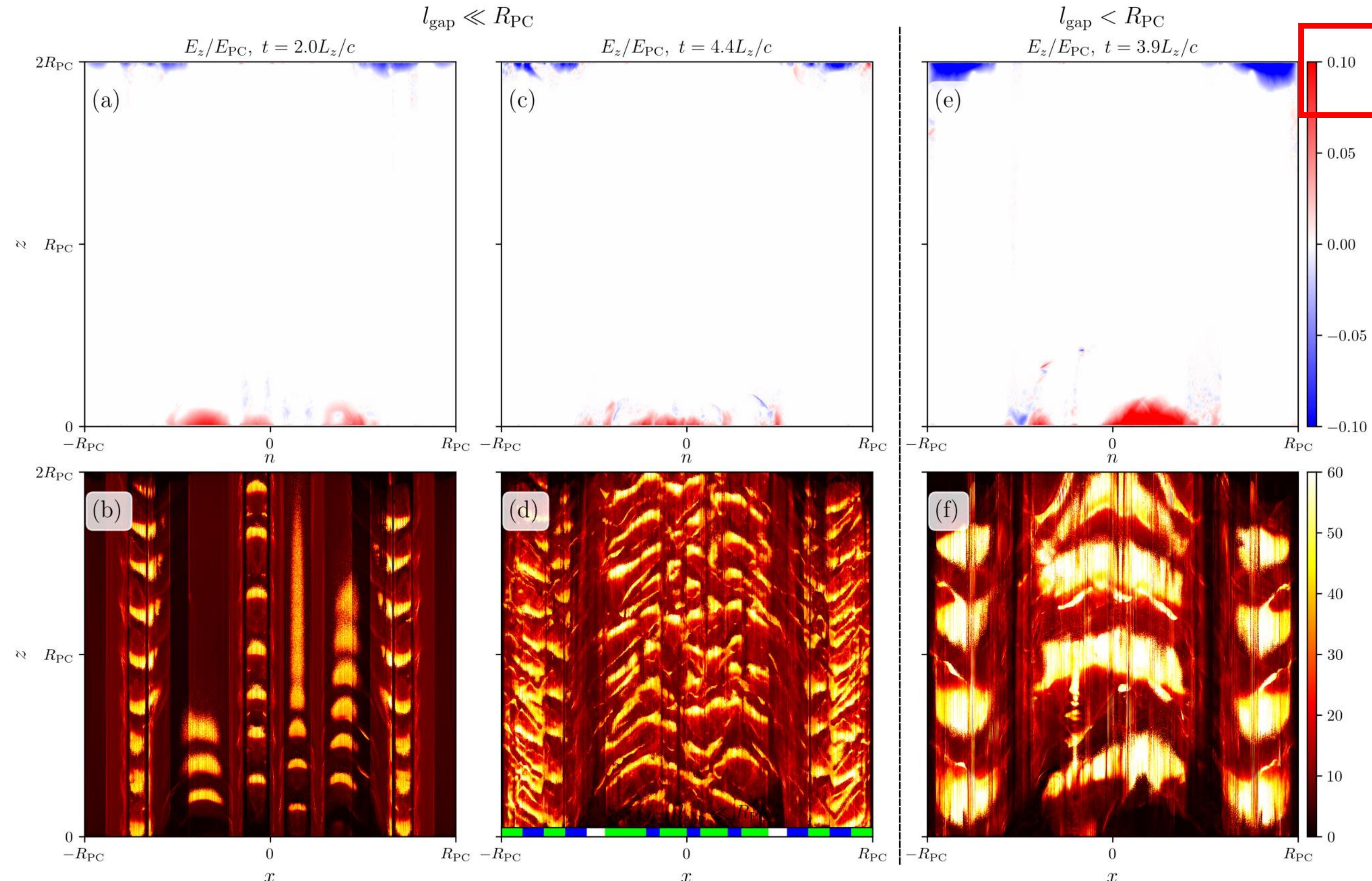
Particle number density:



Longer cyclic period.

## (2) RS model: generally similar to SCLF, but with larger electric field for $j/j_G > 1$

$E_{\parallel}$  field:

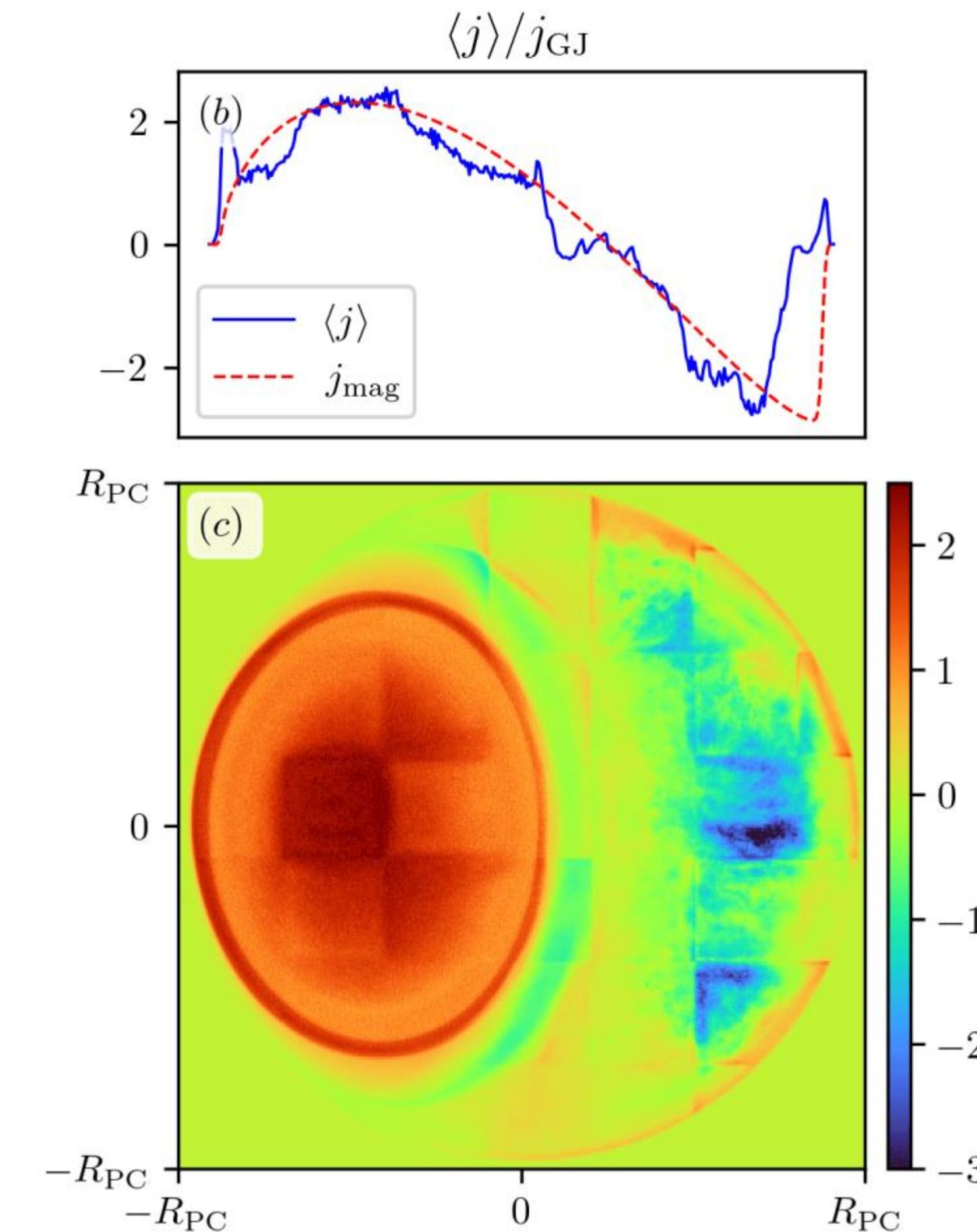
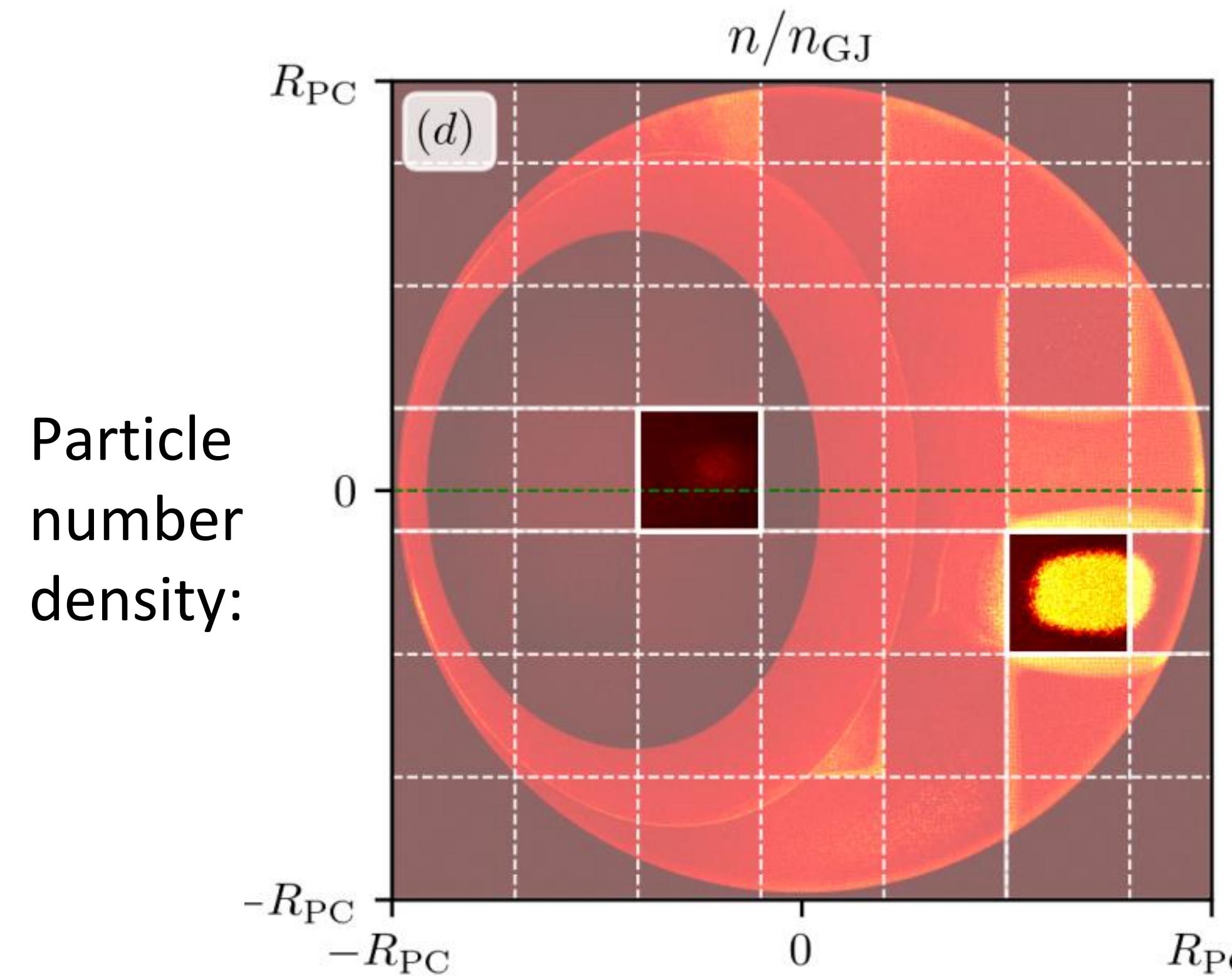


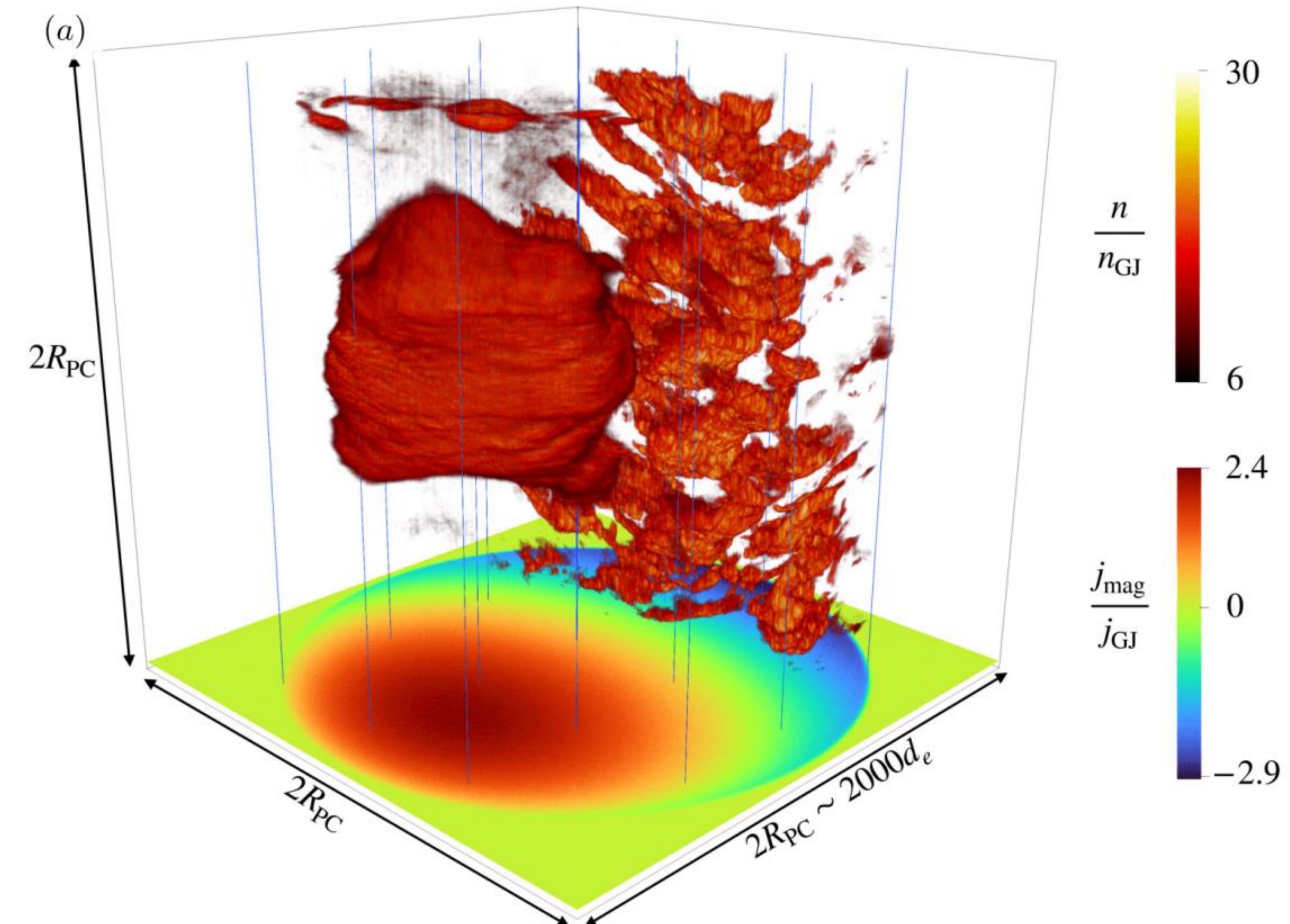
Particle number density:

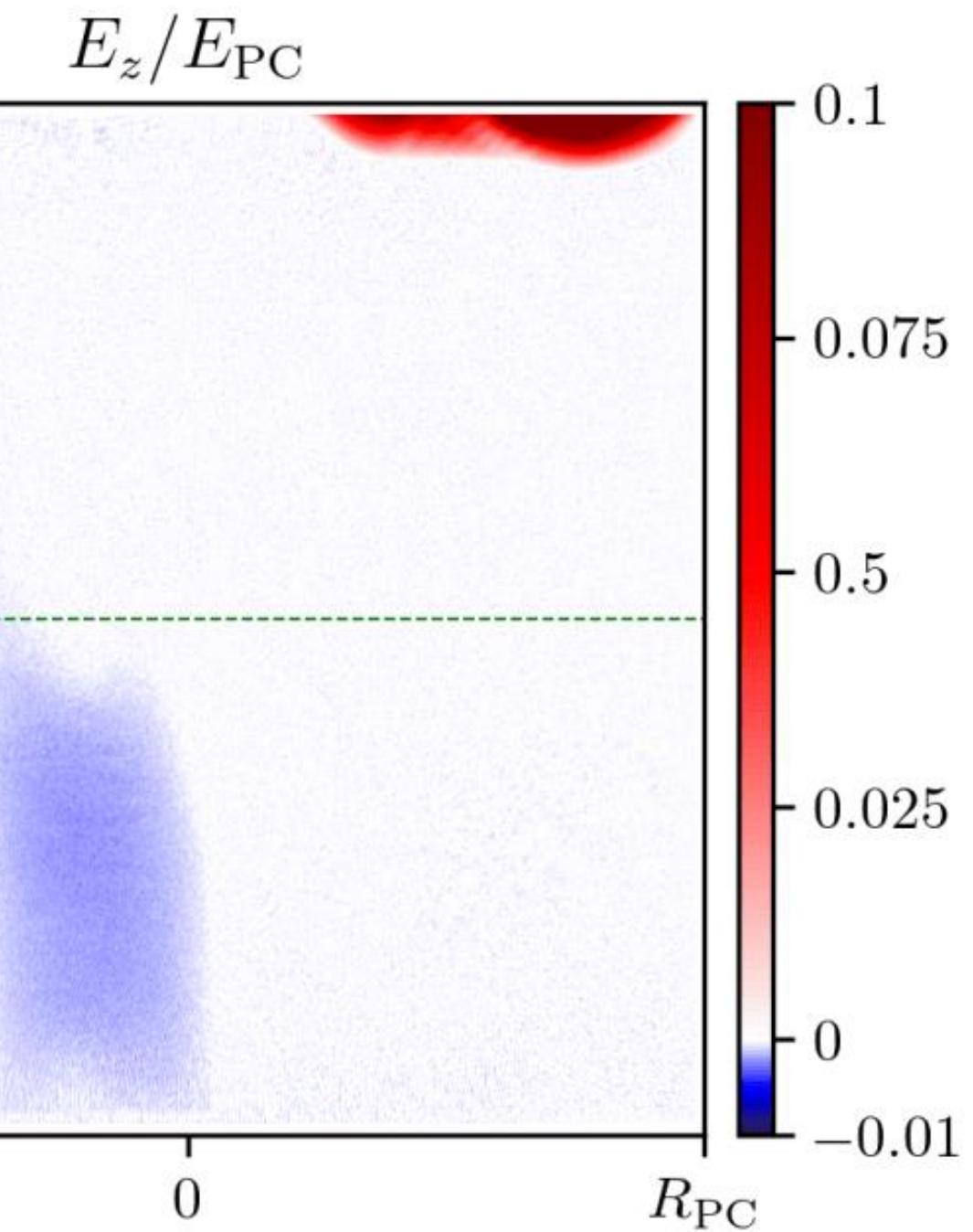
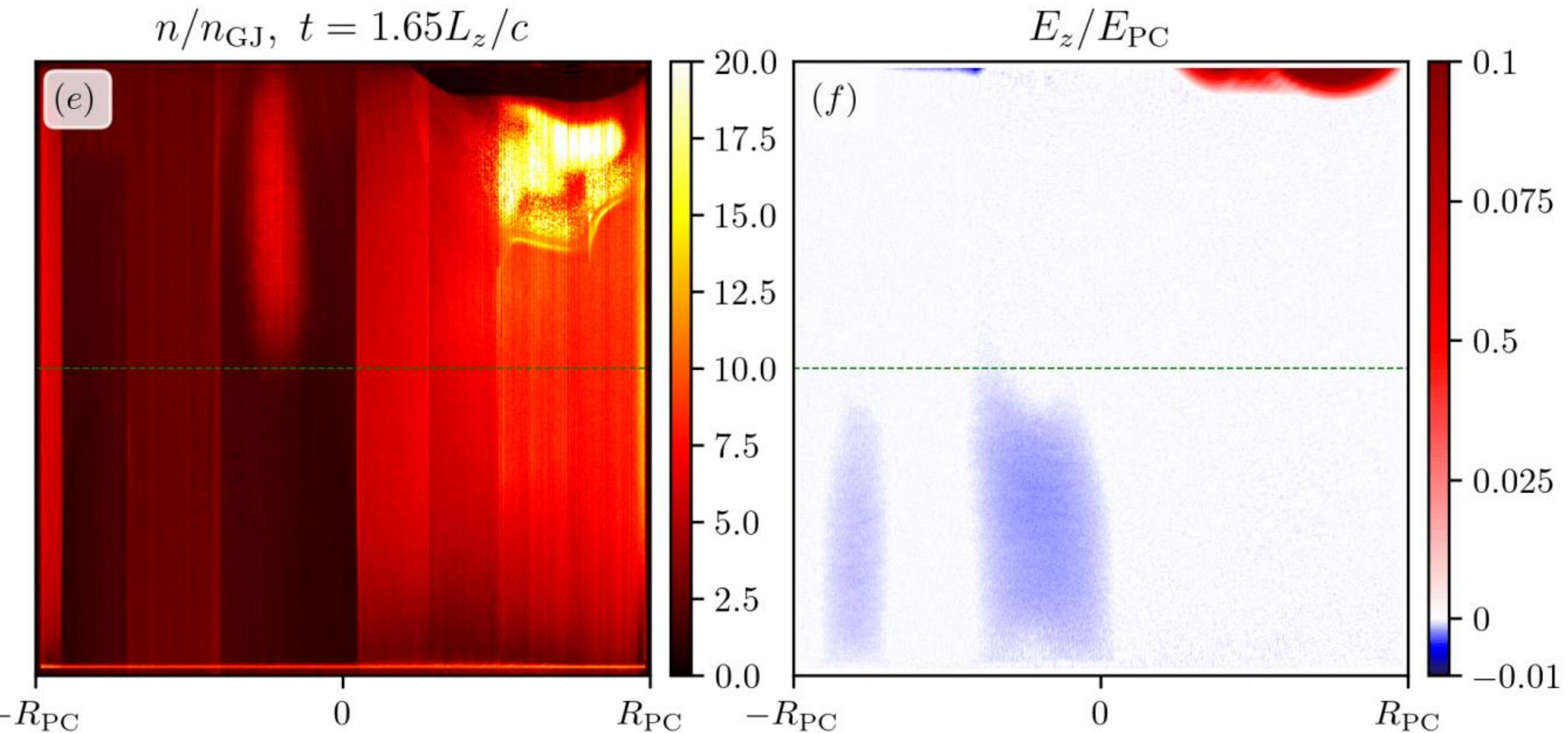
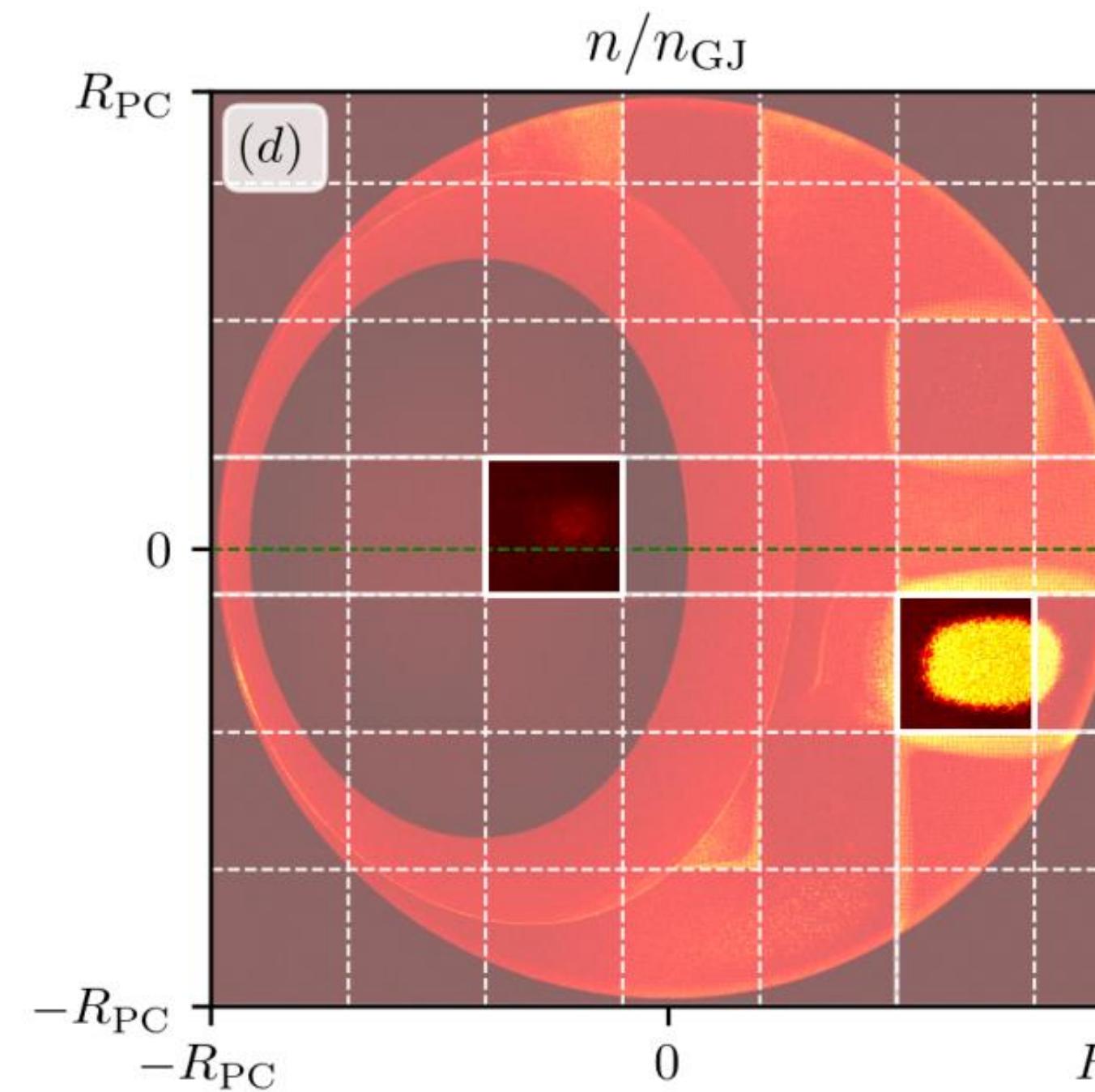
### (3) 3D, SCLF

60° inclined rotator. Small gap for  $j/j_{\text{GJ}} < 0$ , larger gap for  $j/j_{\text{GJ}} > 1$ .

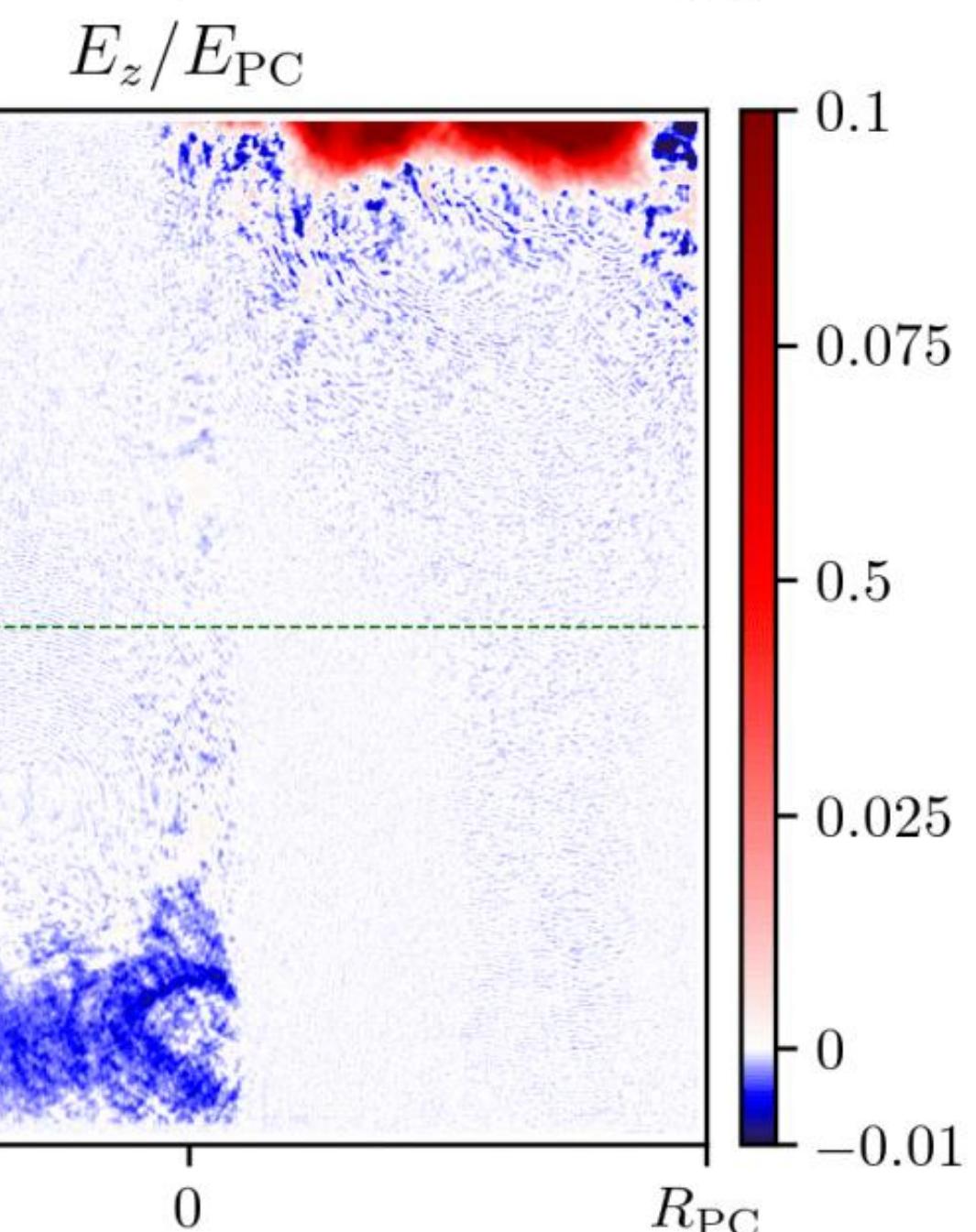
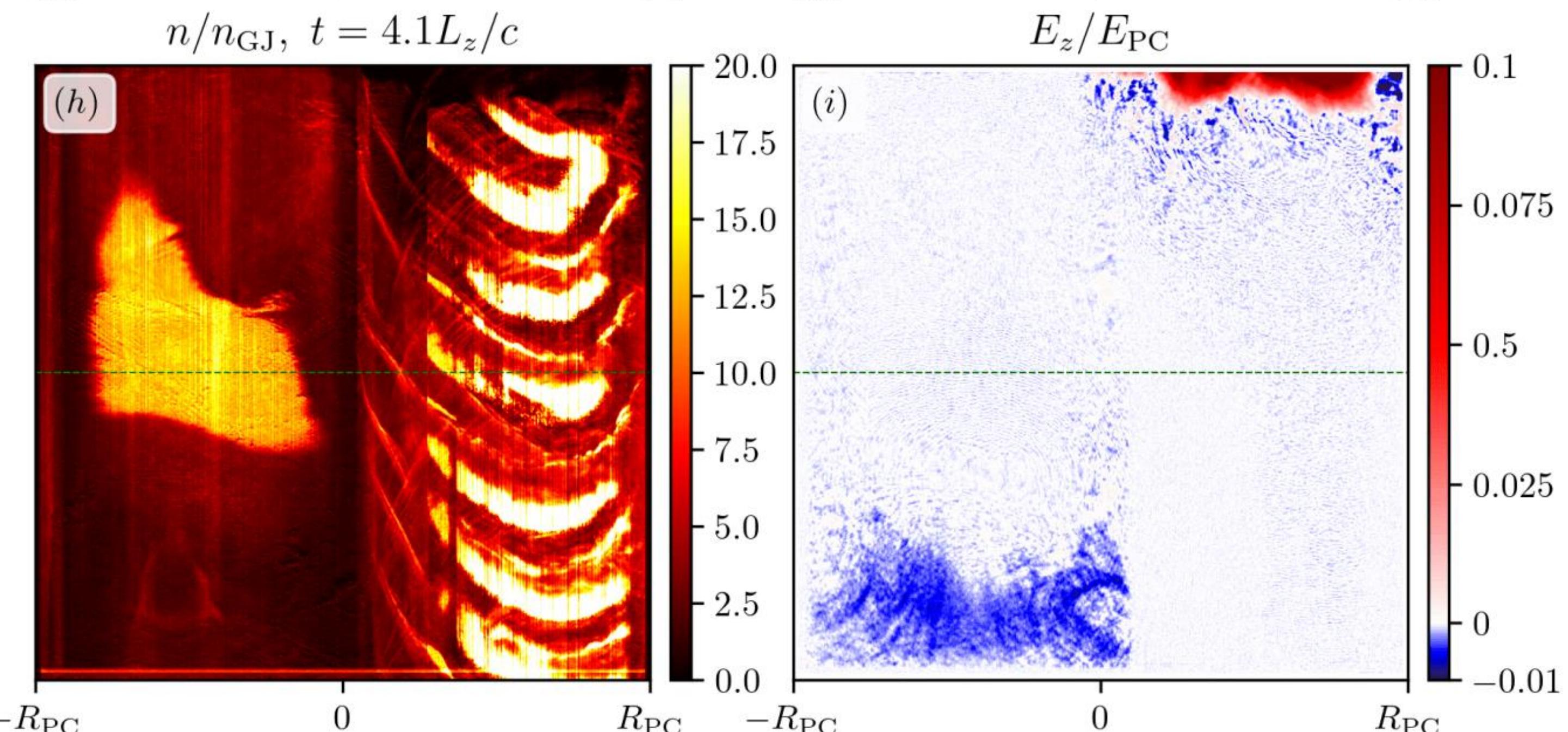
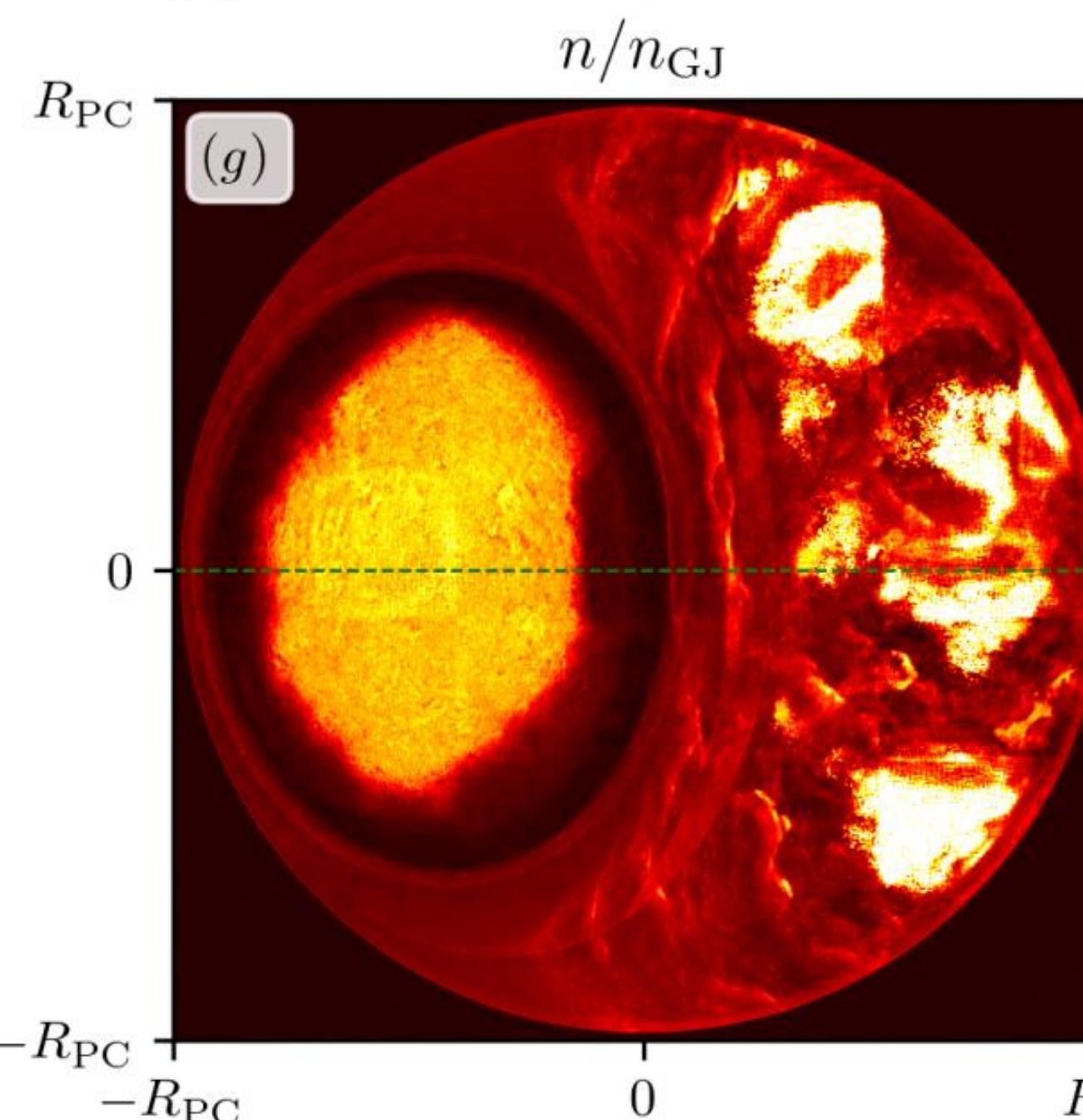
Divide polar cap into  $6 \times 6$  square patches. Two patches with initial plasma injection.







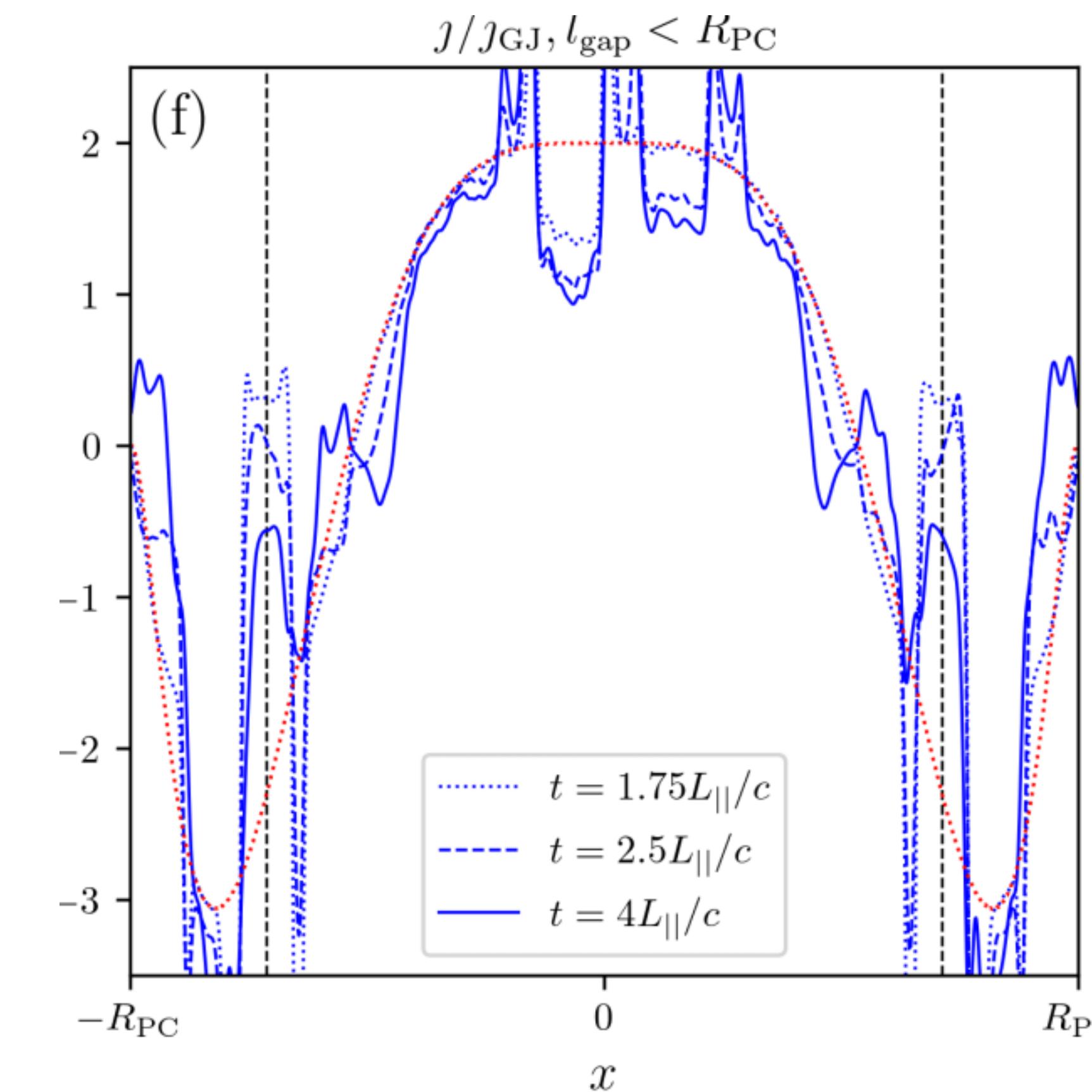
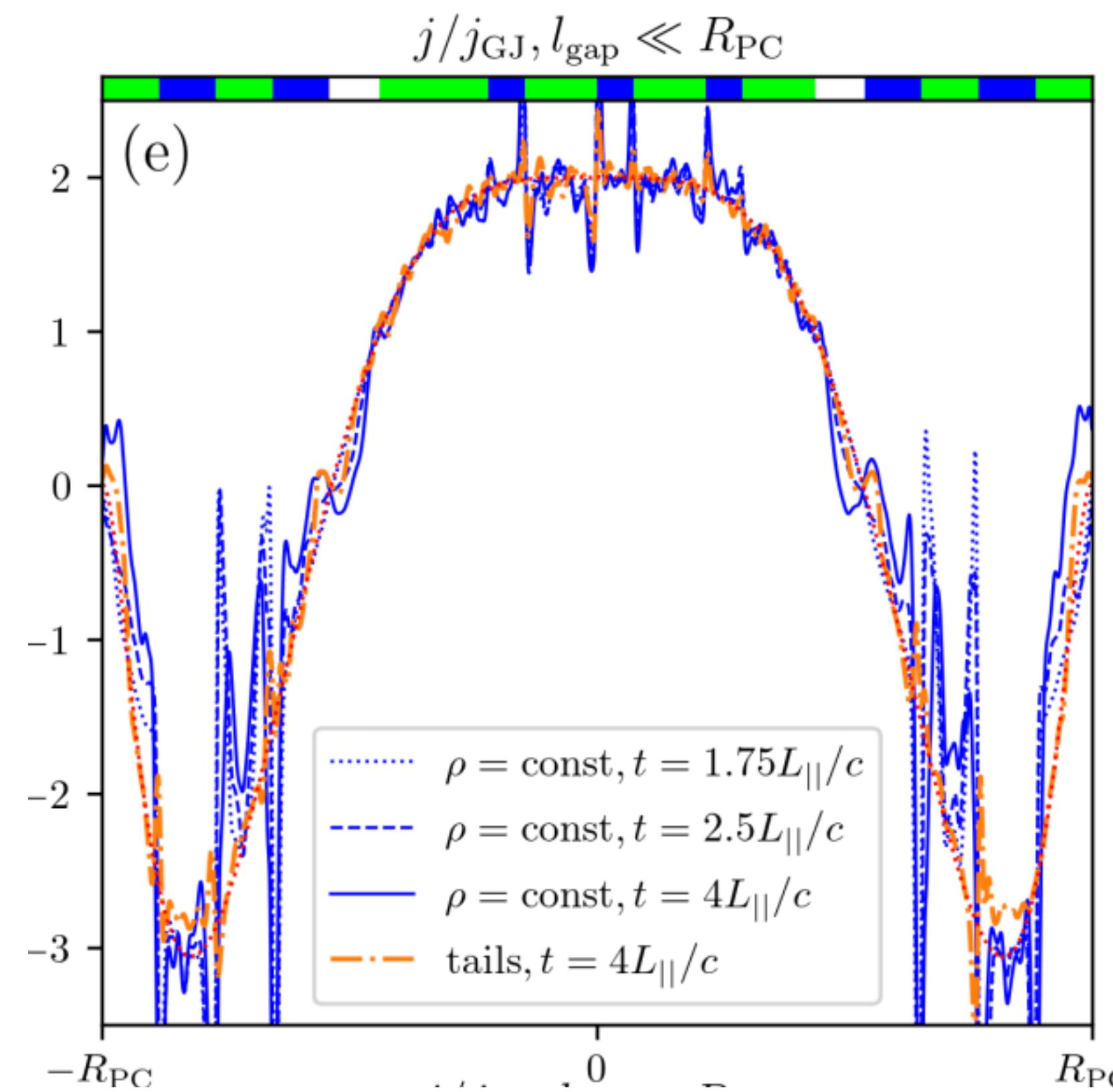
Early time



Late time

## (4) Check for the model validity: evolution of magnetic field twist $\longleftrightarrow$ evolution of magnetospheric current

Large gaps lead to a noticeable untwist of the field lines.

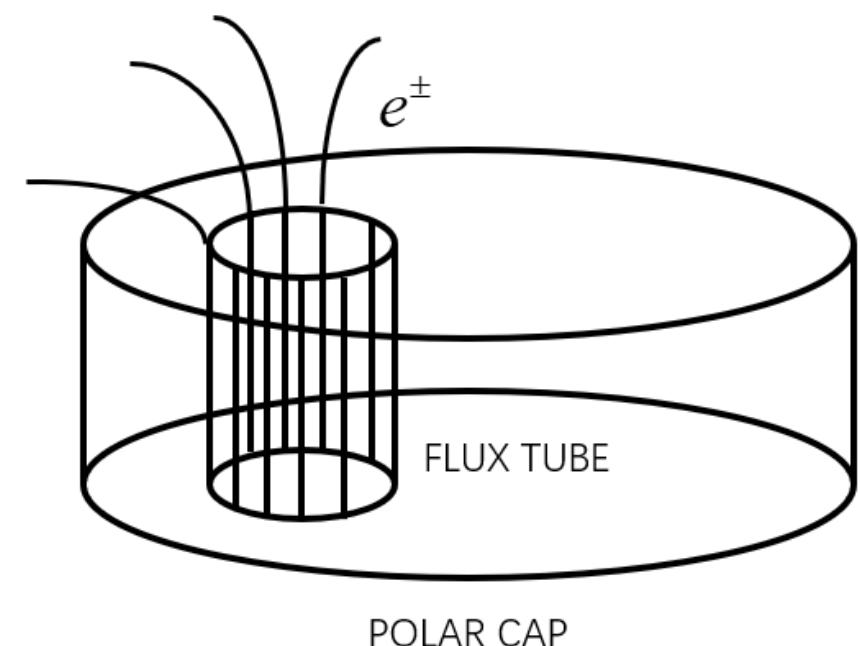


# IV. Conclusion & Discussion

Main conclusion: transverse coherence scale of a discharge zone  $\sim$  longitudinal gap size

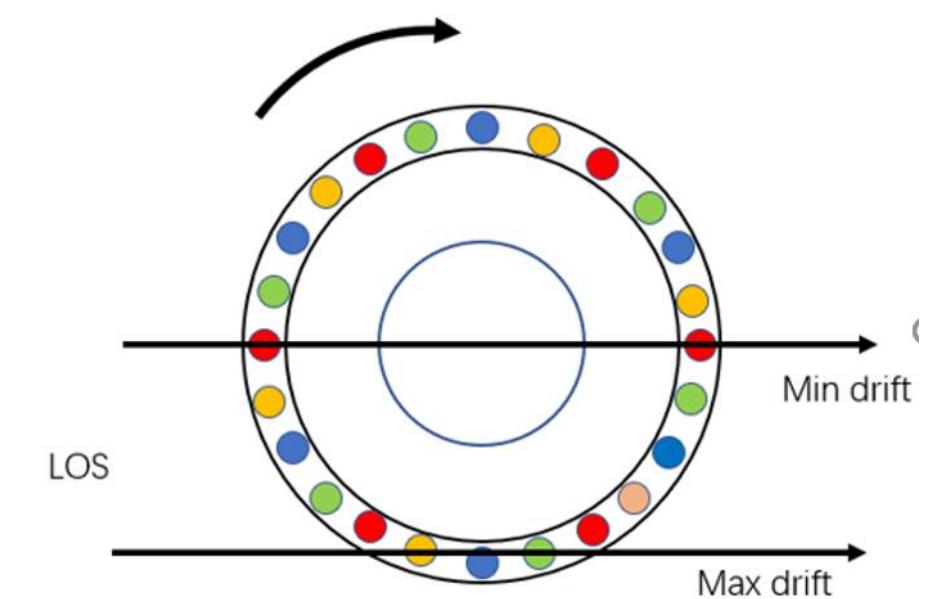
Discussion point 1: **NO spark.**

Polar caps are filled with discharge regions.  
NO noticeable plasma drifting.



Single pulse timescale  $\gg$  discharge timescale

Single pulse modulation  $\leftarrow$  Radiation happens at discharge boundaries?



Discussion point 2: for old pulsars, plasma density may be smaller → deviate from FFE.  
→ may have different properties from this paper's simulation.  
Larger gap → significant twist (at light cylinder) evolution  
→ larger timescale evolutions  
→ nulling... in old pulsars?

Discussion point 3: repetition rate of discharge... too artificial?

感谢大家 Thanks for your attention.