Bunching mechanism for coherent curvature radiation in pulsar magnetospheres. ApJ.

Importance: Radio radiation by pulsars needs to be coherent ⇒ bright enough
⇑
Particle bunching.

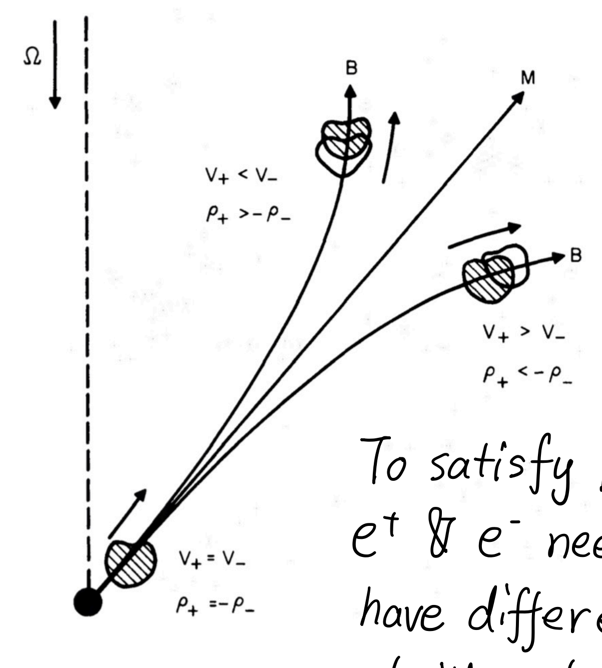
Basic assumptions:

(i) Magnetosphere almost follows ρ_{GJ} (Goldreich & Julian 1969)

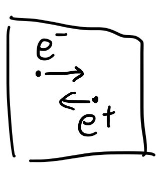
(ii) Radio radiation particles are $e^{\pm}$

(iii) Radio radiation particles originate from $\gamma+\gamma\rightarrow e^{\pm}$, where HE γs are emitted by primary particles near pulsar surface.

Basic picture:



To satisfy $\rho=\rho_{GJ}$, e^+ & e^- need to have different velocities along $\vec{B}$.



(in a certain frame)

⇒ Two stream instability (we'll discuss later)
⇒ particles' bunching.

Equations:

(a). Starting from aligned rotators

$$\begin{cases}\vec{E}+\vec{\beta}\times\vec{B}=0 \quad (\text{ideal MHD}) \\ \vec{\beta}=k\vec{B}+(\Omega r/c)\sin\theta\cdot\hat{\phi}\end{cases} \quad (\text{GJ } 1969)$$

$$\Rightarrow \rho_{GJ}=\frac{\nabla\cdot\vec{E}}{4\pi}=-\frac{\vec{\Omega}\cdot\vec{B}}{2\pi c}\left[1-\left(\frac{\vec{\Omega}\times\vec{r}}{c}\right)^2\right]^{-1}$$

$$=-\frac{\vec{\Omega}\cdot\vec{B}}{2\pi c}\cdot f \quad . \quad f=1+O\left(\frac{\Omega^2r^2}{c^2}\right)$$

↑ Balanced magnetosphere

Actual density: consists of several species

$$\rho=\sum_i \rho^{(i)}$$

When there's radiation particles: $\vec{j}_{\parallel}$ ($/\!/\vec{B}$) arises

If no particle production along some segment of a field line:

$$\frac{\partial\rho}{\partial t}+\nabla\cdot\vec{J}=0 \Rightarrow \nabla\cdot\vec{J}=0 \Rightarrow \nabla\cdot\vec{j}_{\parallel}^{(i)}=0$$

$$\Rightarrow \vec{j}_{\parallel}^{(i)}=-\frac{\alpha^{(i)}\Omega}{2\pi}\cdot\vec{B} \quad , \quad \alpha^{(i)}=\text{Const (along certain field line)}$$

In an active radio psr, there's 3 species:

- beam plasma particles (primary particles): $\gamma_B\sim10^6$
- electrons, positrons: $\gamma_{\pm}\sim10^2$

Write their $\vec{J}$ & ρ s:

$$\vec{J}_B=\rho_B c\cdot\hat{B}=-\frac{\alpha\Omega\vec{B}}{2\pi} \quad , \quad \rho_B=-\frac{\alpha\Omega B}{2\pi c}$$

$$\vec{J}_{\pm}=\rho_{\pm}v_{\pm}\hat{B}=-\frac{\alpha_{\pm}\Omega\vec{B}}{2\pi} \quad , \quad \rho_{\pm}=-\frac{\alpha_{\pm}\Omega B}{2\pi c}\cdot\frac{c}{v_{\pm}}$$

$$\rho=\rho_B+\rho_++\rho_-=-\frac{\vec{\Omega}\cdot\vec{B}f}{2\pi c} \quad (=\rho_{GJ})$$

At $e^{\pm}$'s birthplace, because $\gamma+\gamma\rightarrow e^{\pm}$, we have:

$$-(\rho_+)_0=(\rho_-)_0 \ ,$$

$$(\rho_B)_0=-\frac{(\vec{\Omega}\cdot\vec{B}\cdot f)_0}{2\pi c} \Rightarrow \alpha=(\hat{\Omega}\cdot\hat{B}f)_0$$

$$\Rightarrow \alpha_{\pm}=\left(\frac{\rho_{\pm}}{\rho_B}\right)_0\left(\frac{v_{\pm}}{c}\right)_0\alpha \quad (\text{Const along a field line})$$

$$1+\left(\frac{\rho_+}{\rho_B}\right)_0\frac{(v_+)_0}{v_+}+\left(\frac{\rho_-}{\rho_B}\right)_0\frac{(v_-)_0}{v_-}=\frac{\hat{\Omega}\cdot\hat{B}\cdot f}{(\hat{\Omega}\cdot\hat{B}\cdot f)_0}$$

$$\Leftrightarrow \left(\frac{\rho_+}{\rho_B}\right)_0\left[\frac{(v_+)_0}{v_+}-\frac{(v_-)_0}{v_-}\right]=\frac{\hat{\Omega}\cdot\hat{B}\cdot f}{(\hat{\Omega}\cdot\hat{B}f)_0}-1$$

$$\frac{(v_+)_0v_--(v_-)_0v_+}{v_+v_-}=\left(\frac{\rho_B}{\rho_+}\right)_0\left[\cdots\right]$$

$v_{\dots}\simeq c$

$$\Rightarrow \frac{v_--v_+}{c}\simeq\left(\frac{\rho_B}{\rho_+}\right)_0\left[\frac{\hat{\Omega}\cdot\hat{B}f}{(\hat{\Omega}\cdot\hat{B}f)_0}-1\right]$$

Plug in $\frac{\rho_{\pm}}{\rho_B}\sim\frac{\gamma_B}{\gamma_{\pm}}$ (Energy conservation...)

$$\Rightarrow \frac{v_--v_+}{c}\sim\pm10^{-4}$$

※ When $|v_--v_+|\Downarrow$, $|\rho-\rho_{GJ}|\Uparrow$, $|\vec{E}\cdot\vec{B}|\Uparrow$

⇒ $|v_--v_+|\Uparrow$

⇒ $|v_--v_+|$ will keep.

So what? $\gamma_{\pm}\rightarrow\gamma_>\sim330$, $\gamma_<\sim70$

Consider wave modes in such system.

Electrostatic waves propagating along $\vec{B}$:

Dispersion relation (Montgomery & Tidman 1964)

$$1=\frac{\omega_<^2}{\gamma_<^3(\omega-ku_<+ik\Delta u_<)^2}+\frac{\omega_>^2}{\gamma_>^3(\omega-ku_>)^2}+\frac{\omega_B^2}{\gamma_B^3(\omega-ku_B)^2}$$

(the last term is crossed out)

$$\left(\omega_<=\sqrt{\frac{4\pi n_<e^2}{m}} \ , \ \omega_>=\sqrt{\frac{4\pi n_>e^2}{m}} \ , \ \omega_B=\sqrt{\frac{4\pi n_Be^2}{m}}\right)$$

↓ Seek solution: real k, complex ω

(pertubations: $n\propto e^{i(kx-\omega t)}\propto e^{(\mathrm{Im}\,\omega)t}$)
(unstable)

$$\mathrm{Re}\,\omega\approx\gamma_<^{1/2}\omega_< \ ,$$

$$\mathrm{Re}\,(\omega/k)\lesssim u_>\lesssim c \ ,$$

$$\frac{\mathrm{Im}\,\omega}{\mathrm{Re}\,\omega}\approx\frac{\omega_>}{\omega_<}\frac{1}{\gamma_<^{1/2}\gamma_>^{3/2}}\approx\frac{1}{\gamma_>^2} \ .$$

equation (13), the mode frequency in the laboratory is

$$\mathrm{Re}\,\omega\approx3\times10^{13}P^{-1/2}B_{12}^{1/2}(R/r)^{3/2}\ \mathrm{rad\ s^{-1}} \ .$$

$R\sim10^6\,\mathrm{cm}$, $r\sim10^{8\text{-}9}\,\mathrm{cm}$ → GHz

ed range near the star, and it drops into the microwav

$$\mathrm{Im}\,\omega\approx3\times10^8P^{-1/2}B_{12}^{1/2}(R/r)^3\ \mathrm{rad\ s^{-1}} \ .$$

→ $10^{3\sim4}\ \mathrm{s^{-1}}$

Growth rate of instability.

Bunching will be established in ms

(for normal pulsar, it works within Light cylinder)

- Discussions:

(1) Aligned rotator ⇒ Oblique rotator

$$\cdots\cdots \ \frac{v_--v_+}{c}=\left(\frac{\rho_B}{\rho_{\pm}}\right)_0\left[\frac{f\Omega_zB_z/B}{(f\Omega_zB_z/B)_0}-1\right] \quad \text{works} \ ✓$$

(2) Spectrum: depend on non-linear saturation of the unstable mode.
$\omega\lesssim\omega_c$ ......

(3) Radiation Reaction: tend to keep bunching (Goldreich & Keeley 1971)