A Theory of Subpulse Polarization Patterns from Radio Pulsars

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(Shunshun Cao)
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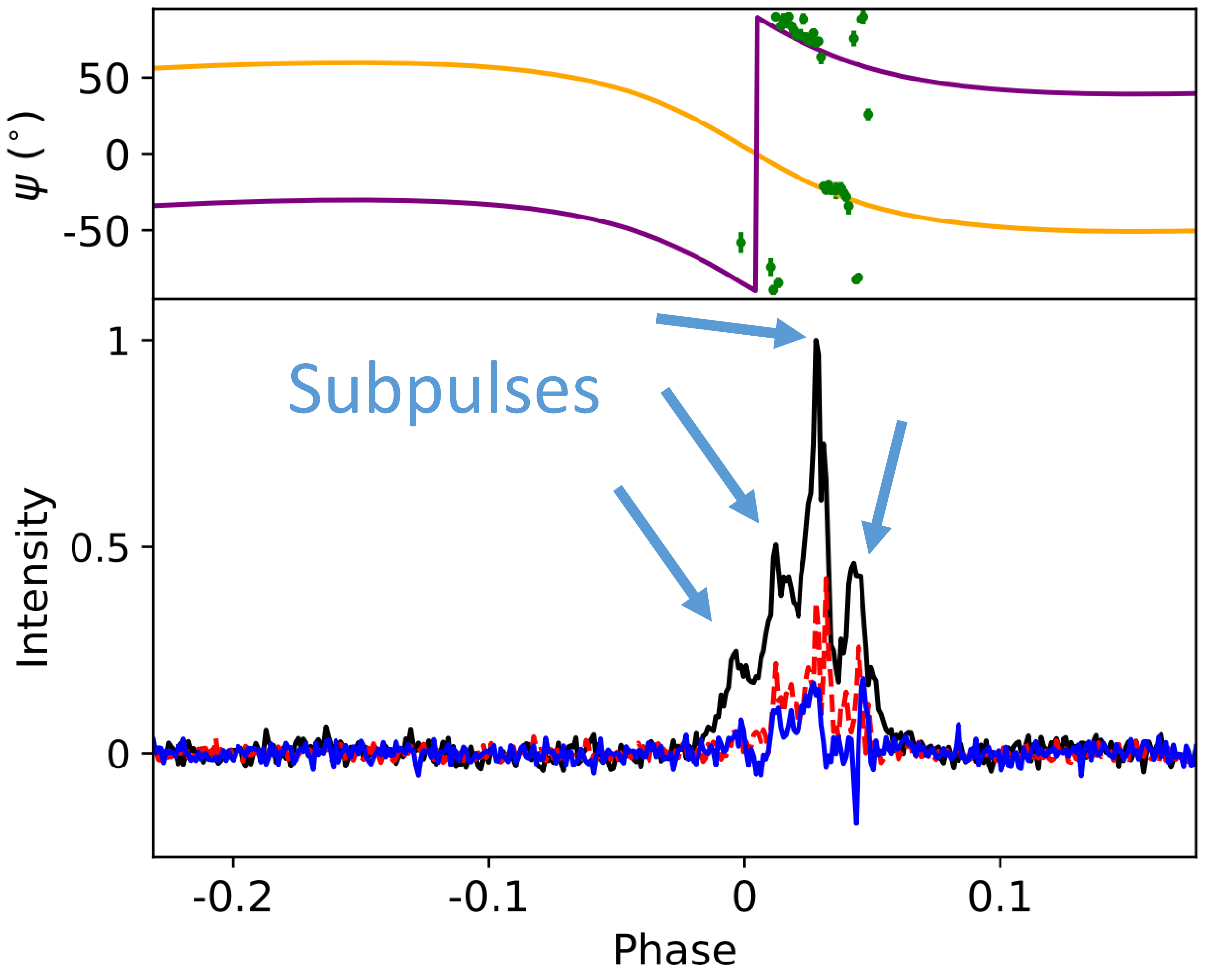
Contents

I. Introduction

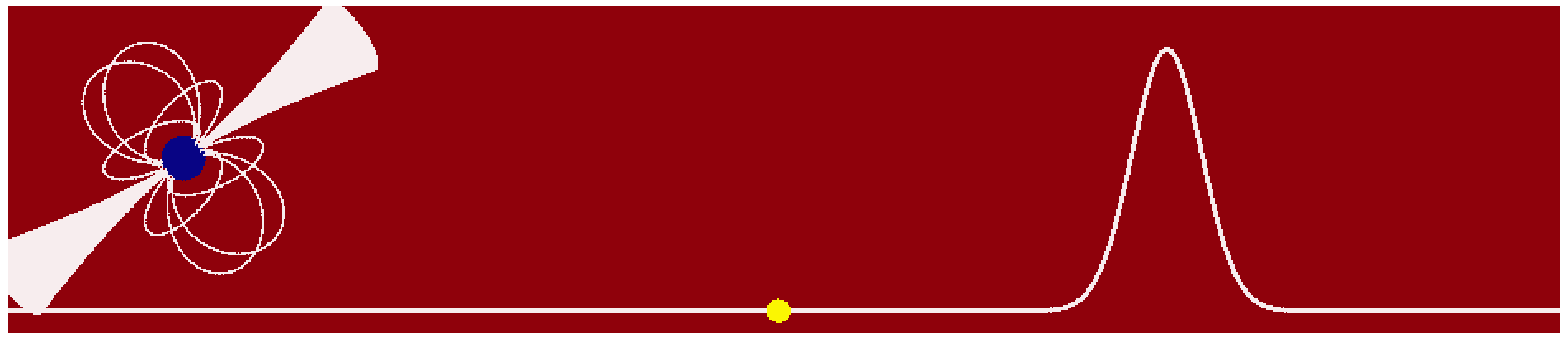
Subpulses in single pulses: various polarization patterns.



e.g., one single pulse of B0943+10.

- ➔Linear polarization
- ➔Circular polarization
- ➔Polarization angle (PA) swing
- ➔PA jump (especially 90° jump)
- ➔Depolarization
- ➔Above phenomena's relation with subpulse position
- ➔…

"A severe burden on any theoretical model"



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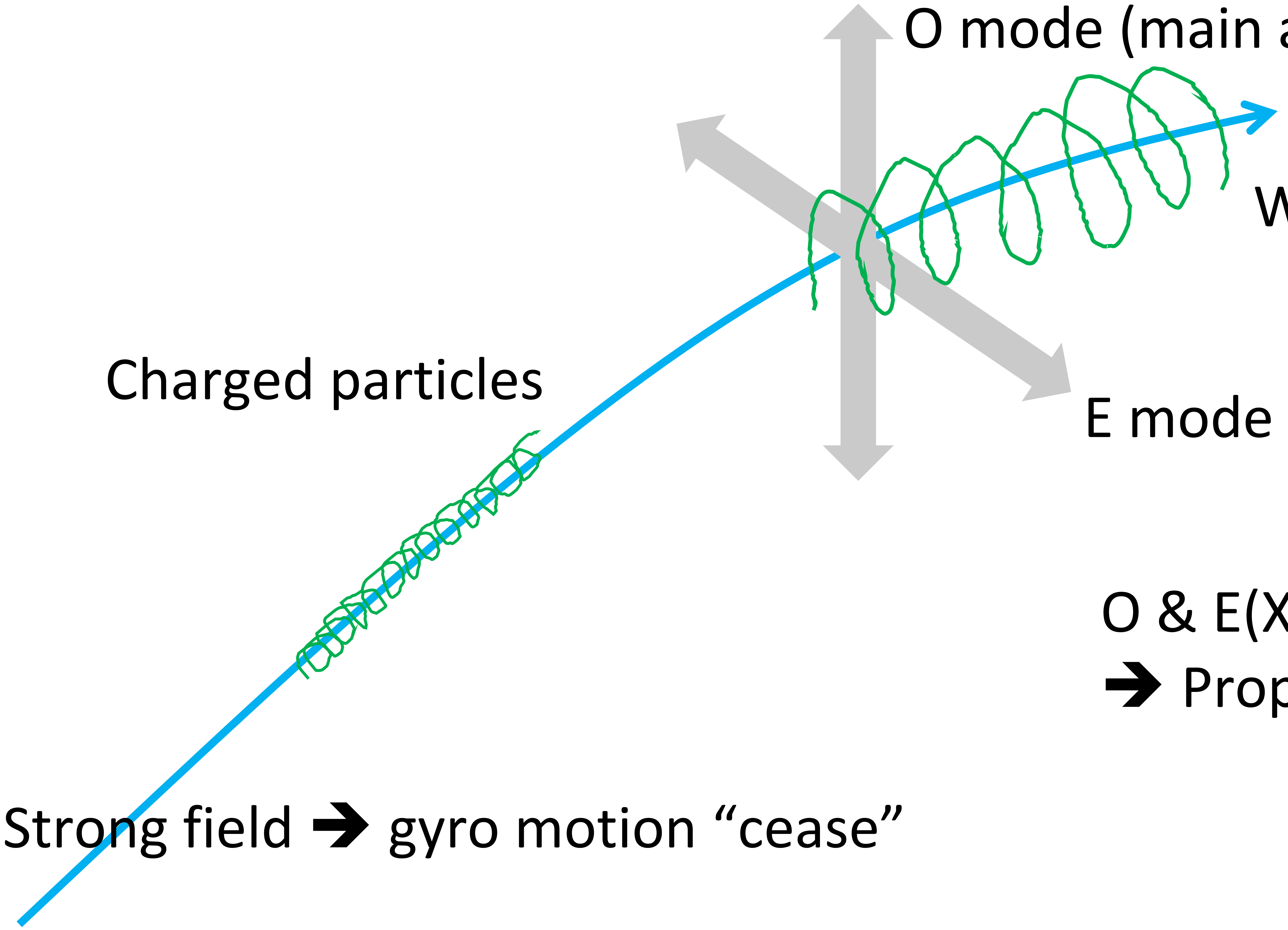
Briefly think: about magnetic fields and electrons/positrons…



Weak field ➔ gyro motion significant

Strong field ➔ gyro motion “cease”

O & E(X) mode: different dispersion relation
➔ Propagate differently.

Curvature radiation & bunching…
➔initial polarization linear/circular…

(1) How radiated?
(2) How propagate?

II. Linearly Polarized Normal Modes

Adiabatic walking: independent propagations of orthogonal modes.

"Adiabatic": $$\left|\lambdabar \frac{\partial}{\partial s} \Delta n\right| \ll |\Delta n|$$

$\Delta n = n_o - n_e$

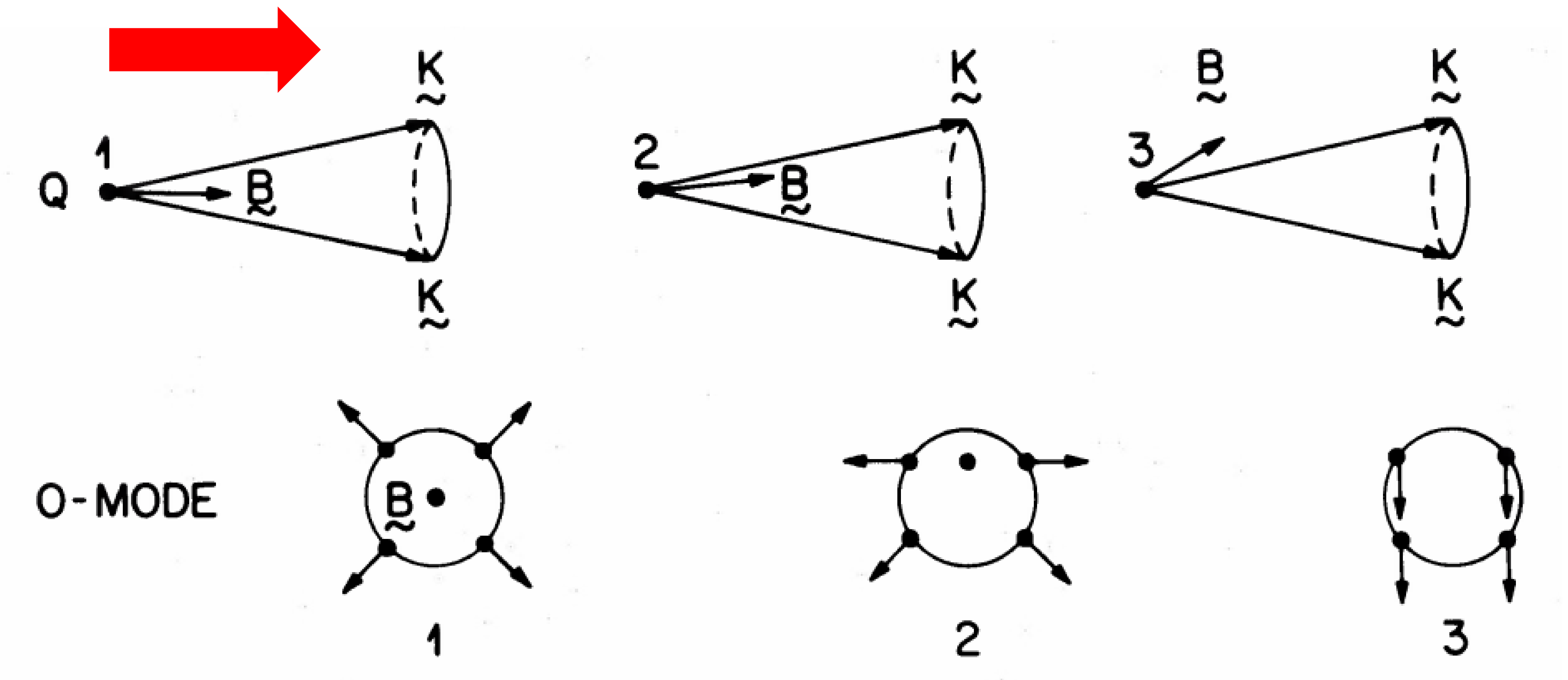
Plasma properties (refraction indices) change slow enough.
➔➔➔ Wave modes are independent.

"Walking": $$\left|\lambdabar \frac{\partial}{\partial s} \phi\right| \ll |\Delta n|$$

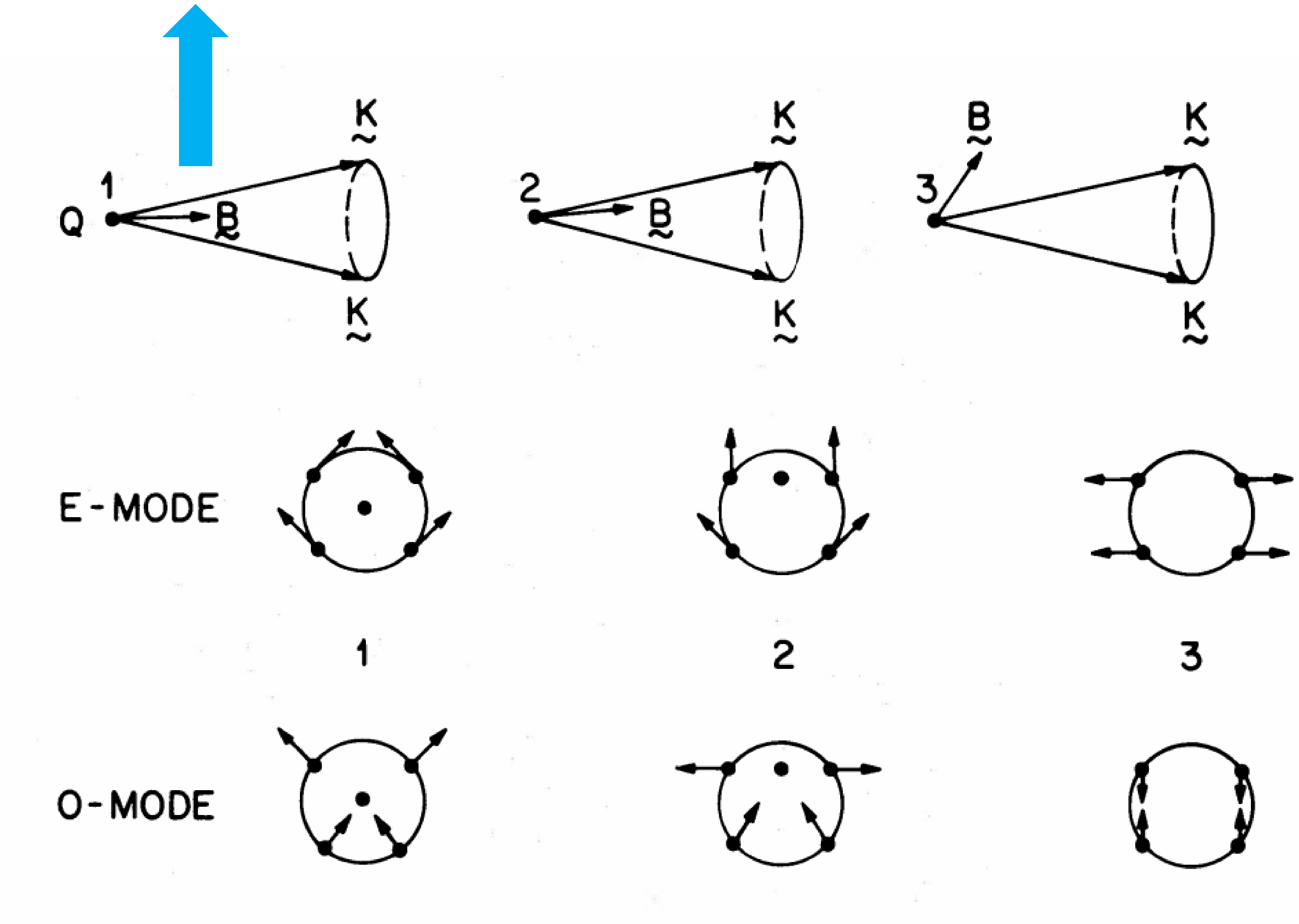
ϕ: PA (or other similar dimensionless parameters about polarization).
Polarization changes slow enough.
➔➔➔ Wave modes polarization **follow** the local magnetic field.
(parallel(O) or perpendicular(E))

Consequences for different ways of acceleration:

(radiation propagates further with local magnetic field changing, if adiabatic walking holds)



O - MODE FROM PARALLEL ACCELERATION



E-MODE AND O-MODE FROM PERPENDICULAR ACCELERATION

$$\theta_w \sim \gamma_w^{-1}$$

Under assumption:

electron/positron distributions are nearly **symmetric** near emission region.

And, $\omega \ll \gamma_w eB/mc$ (so, all linear polarization…)

Question I: how good is adiabatic walking near emission? ➔ needs some quantitative analysis.

Expression for Δn (Melrose & Stoneham 1977):

$$\Delta n = \frac{\omega_p^2 \sin^2\theta}{2\omega^2\gamma^3(1-\beta\cos\theta)^2} f(\omega)\,, \qquad (4)$$

with

$$\beta \equiv v/c\,, \qquad \omega_p^2 = 4\pi\bar{n}e^2/m\,, \qquad (5)$$

and

$$f(\omega) \equiv \frac{(eB/mc)^2}{(eB/mc)^2 - \gamma^2\omega^2(1-\beta\cos\theta)^2}\,. \qquad (6)$$

Particle num density: $\bar{n} \propto r^{-3}$

θ = <k,B>, ignore refraction

$\theta \sim \gamma_w^{-1}$

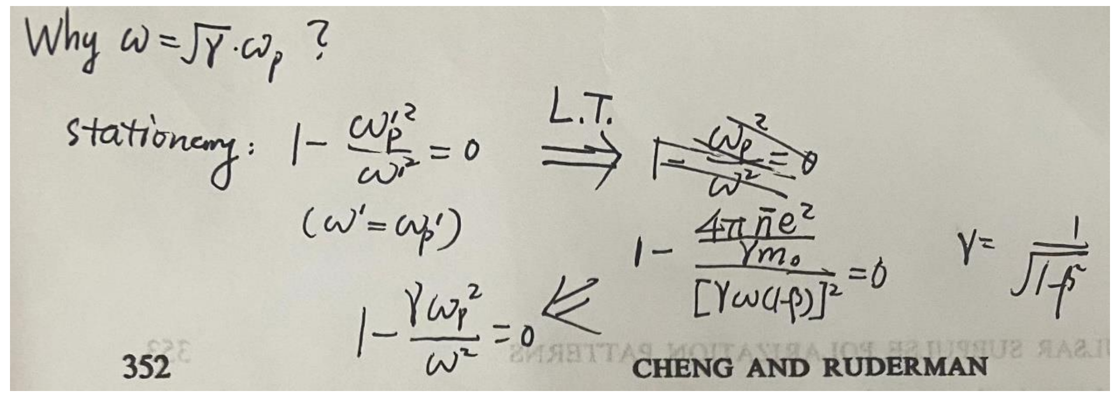
If only a single γ:

$$\Delta n \sim \frac{2}{\gamma_w^4(\theta^2 + \gamma_w^{-2})}\left(\frac{r_E}{r}\right)^3$$

Near emission region

$$\Delta n \sim 1/\gamma^2$$

Radiation freq (from bunching): $\omega^2 = \left[\gamma_w \omega_p^2\right]_e$

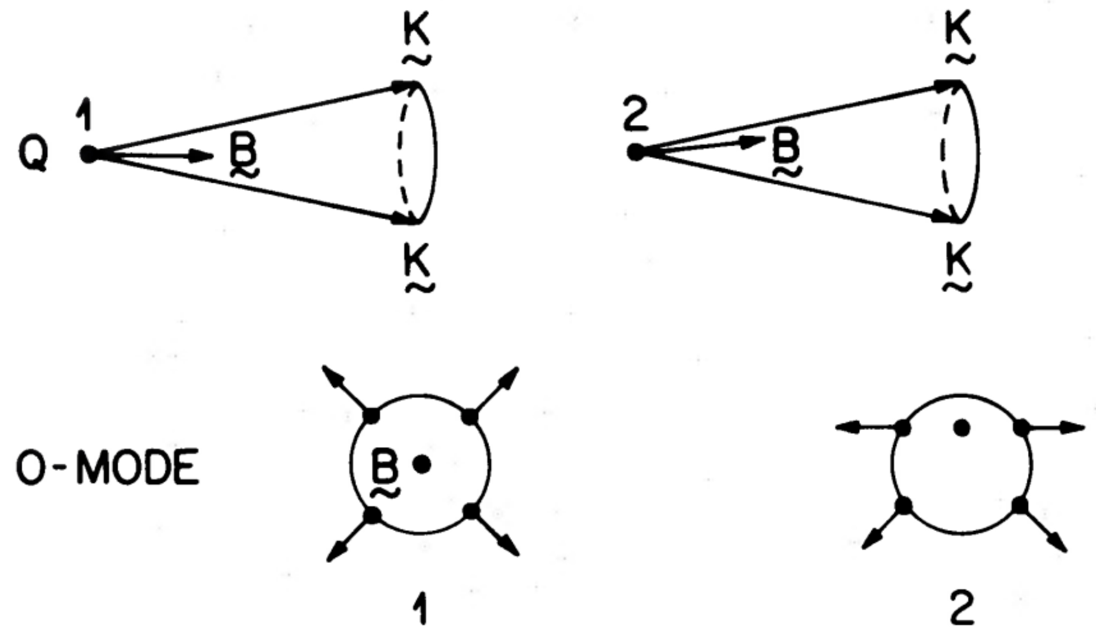


Question I: how good is adiabatic walking near emission? ➔ needs some quantitative analysis.

"Adiabatic": $|\lambda\!\!\!{}^{-} \frac{\partial}{\partial s} \Delta n| \ll |\Delta n|$

"Walking": $\left| \lambda\!\!\!{}^{-} \frac{\partial}{\partial s} \phi \right| \ll |\Delta n|$

$\lambda\!\!\!{}^{-} / r_E \ll 1$



$$\frac{d\phi}{ds} \sim \gamma_w \kappa$$

$\kappa \sim (r r_{1c})^{-1/2}$

curvature

$$\omega > \gamma_w{}^3 c \kappa$$

Curvature radiation critical frequency

Particle num density: $\bar{n} \propto r^{-3}$

θ = <k,B>, ignore refraction

$\theta \sim \gamma_w{}^{-1}$

If only a single γ:

$$\Delta n \sim \frac{2}{\gamma_w{}^4(\theta^2 + \gamma_w{}^{-2})} \left(\frac{r_E}{r}\right)^3$$

Near emission region

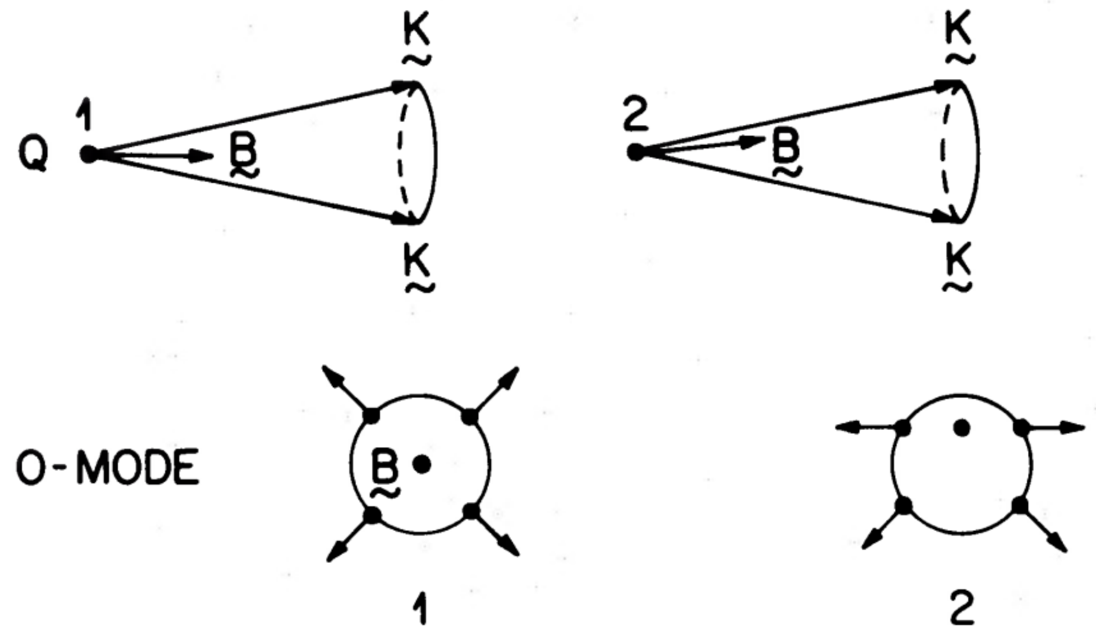
$$\Delta n \sim 1/\gamma^2$$

Question I: how good is adiabatic walking near emission? ➔ needs some quantitative analysis.

"Adiabatic": $|\lambda\!\!\!^{-} \frac{\partial}{\partial s} \Delta n| \ll |\Delta n|$

"Walking": $\left| \lambda\!\!\!^{-} \frac{\partial}{\partial s} \phi \right| \ll |\Delta n|$

$\lambda\!\!\!^{-}/r_E \ll 1$



$\frac{d\phi}{ds} \sim \gamma_w \kappa$

$\kappa \sim (r r_{1c})^{-1/2}$

curvature

$\omega > {\gamma_<}^3 c\kappa$

Easier to satisfy

Particle num density: $\bar{n} \propto r^{-3}$

θ = <k,B>, ignore refraction

$\theta \sim {\gamma_w}^{-1}$

If two γs:

$\gamma_< < \gamma_w$

$\gamma_> > \gamma_w$

$$\Delta n \sim \frac{2}{\gamma_w {\gamma_<}^3 (\theta^2 + {\gamma_w}^{-2})} \left(\frac{r_E}{r}\right)^3$$

Near emission region

$\Delta n \sim 1/\gamma^2$

Question II: parallel or perpendicular acceleration? ➔ energy (power) comparison.

$$P = \frac{2q^2}{3c^3}\mathbf{a}'\cdot\mathbf{a}' = \frac{2q^2}{3c^3}\left(a_{\parallel}'^2 + a_{\perp}'^2\right) = \frac{2q^2}{3c^3}\gamma^4\left(a_\perp^2 + \gamma^2 a_\parallel^2\right).$$

Rybicki & Lightman
Radiative processes in Astrophysics

Parallel acc ("pure" bunching radiation):
(in particle originally rest frame)

$$\hat{a}_\parallel \sim c\omega/\gamma_w$$

$$\Rightarrow \quad P_\parallel \sim \frac{Q^2\omega^2}{c\gamma_w{}^2}$$

Perpendicular acc (curvature radiation):

$$P_\perp \sim Q^2\kappa^2\gamma_w{}^4 c \sim \frac{\omega_{\mathrm{curv}}{}^2 Q^2}{\gamma_w{}^2 c}$$

$$P_\perp \gtrless P_\parallel ,$$

$$\omega_{\mathrm{curv}} \gtrless \omega .$$

$\omega > \omega_{\mathrm{curv}}$ ➔➔ pure O mode

$\omega < \omega_{\mathrm{curv}}$ ➔➔ O & E mode, E dominate, O suppressed

(Razin effect, $n_o<n_e\sim1$, $\theta_b \sim \sqrt{1-n_r^2\beta^2}$)

III. Orthogonal Mode Transitions

(1) **Incoherent** mixture of two polarized beams

Horizontal axis:

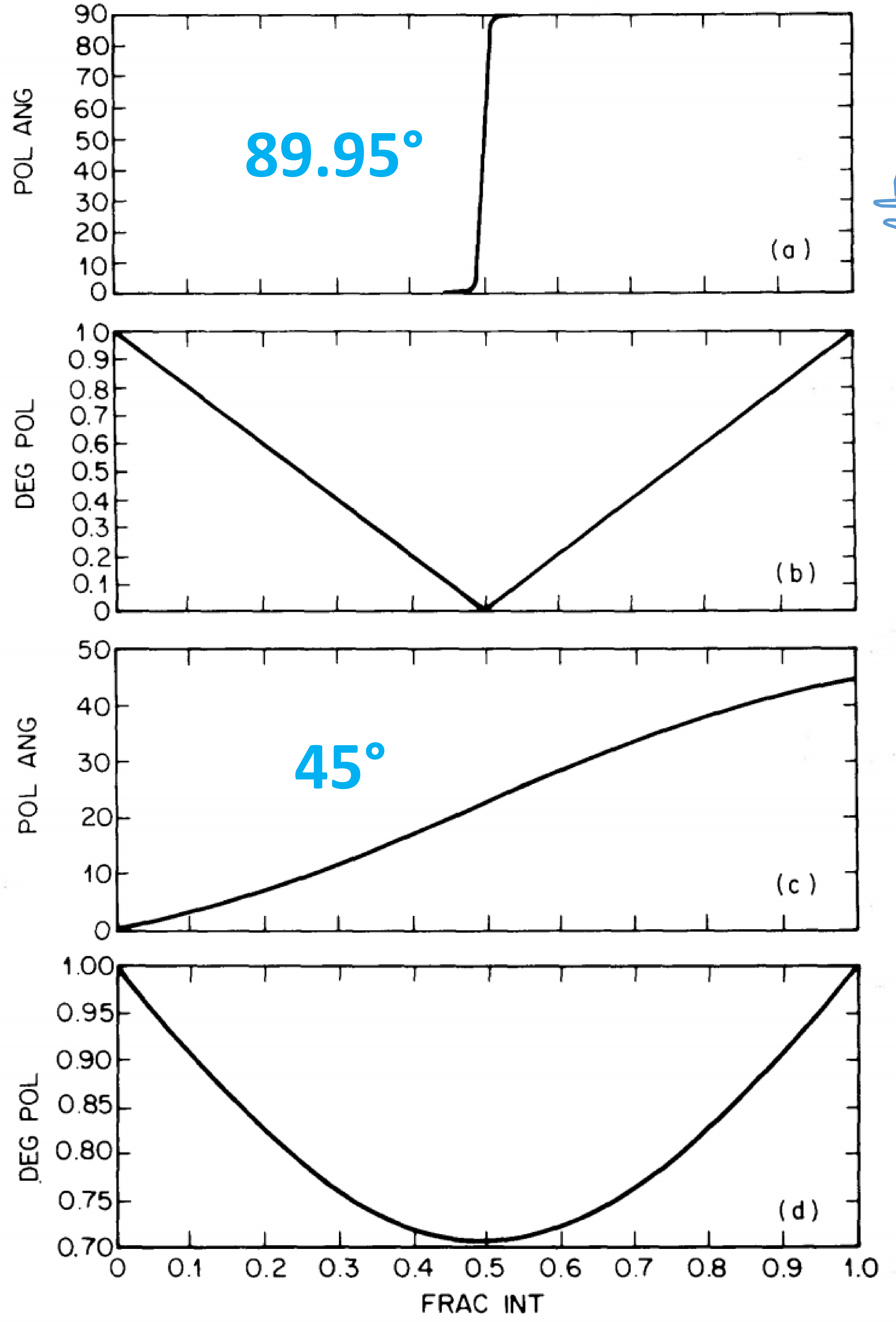
$$I_2/(I_1 + I_2)$$

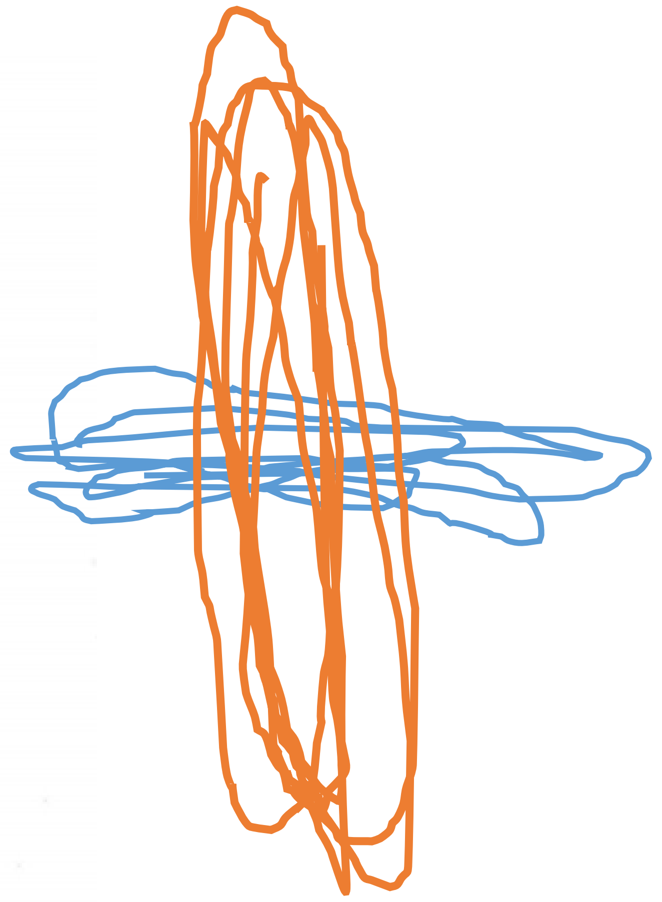
$\sim$ Rotation phase

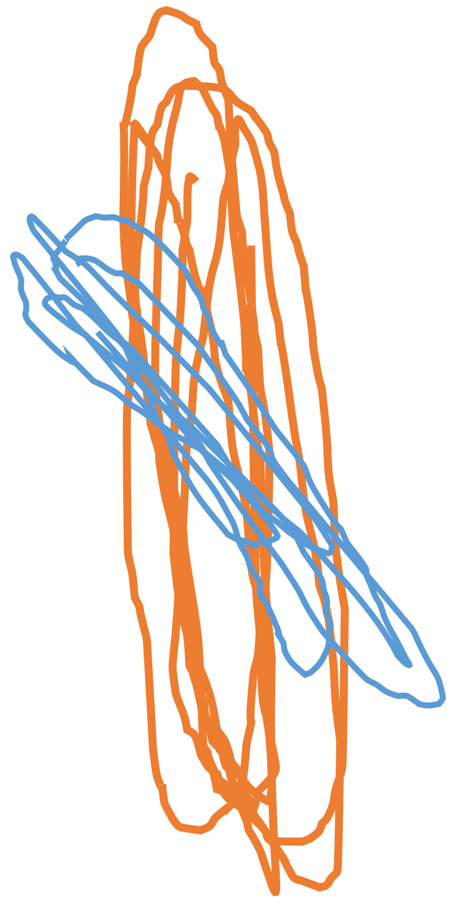
Two orthogonal polarized incoherent beams:

➔sudden jump when $I_1 = I_2$

➔low polarization degree.







(2) Parallel acceleration v.s. perpendicular acceleration

Recall that:

$$P_\perp \gtrless P_\parallel ,$$

$$\omega_{\rm curv} \gtrless \omega .$$

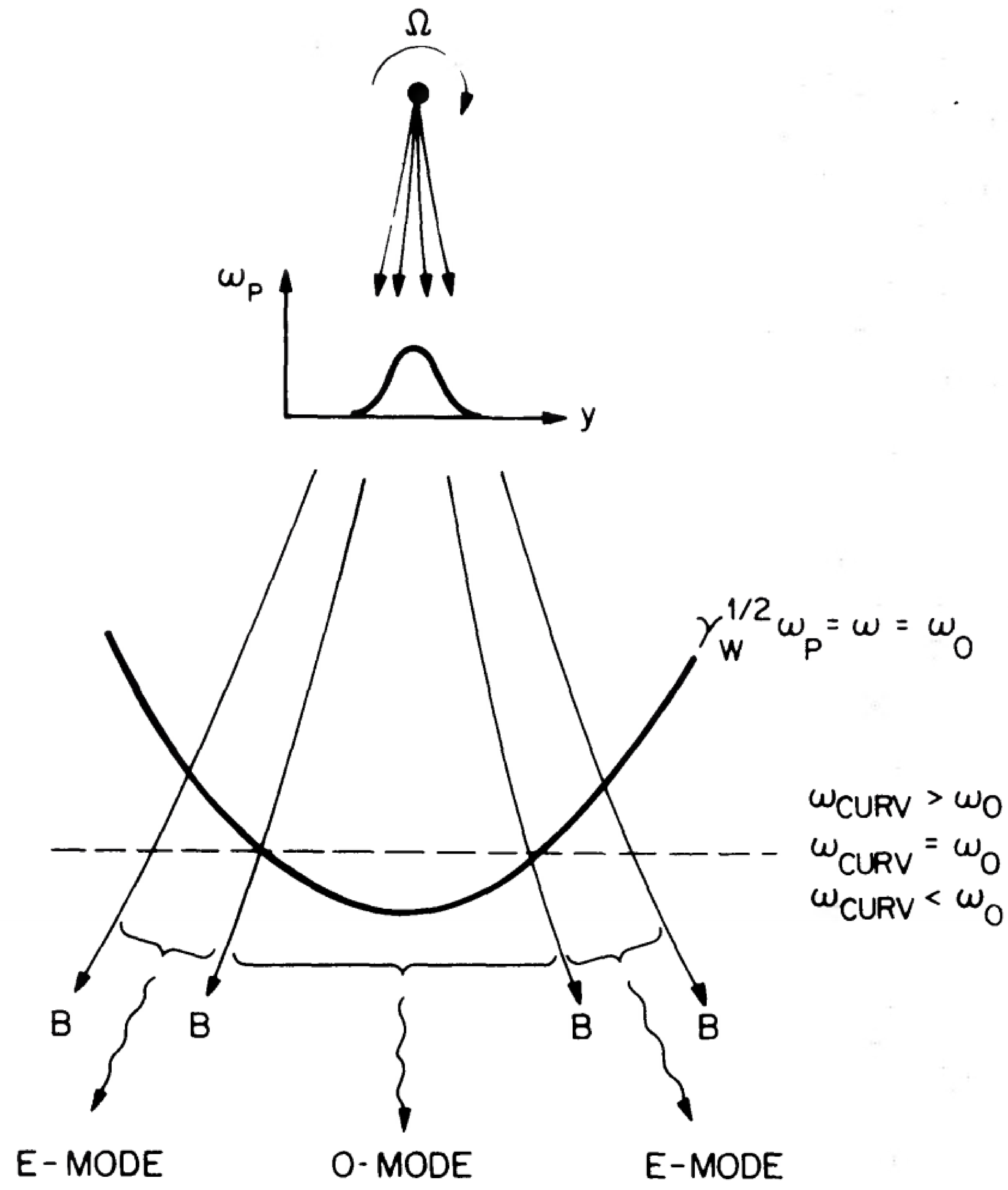
$$\omega_{\rm curv} = \gamma_w{}^3 c\kappa$$

$$\omega^2 = (\gamma_w \omega_p{}^2)_e$$

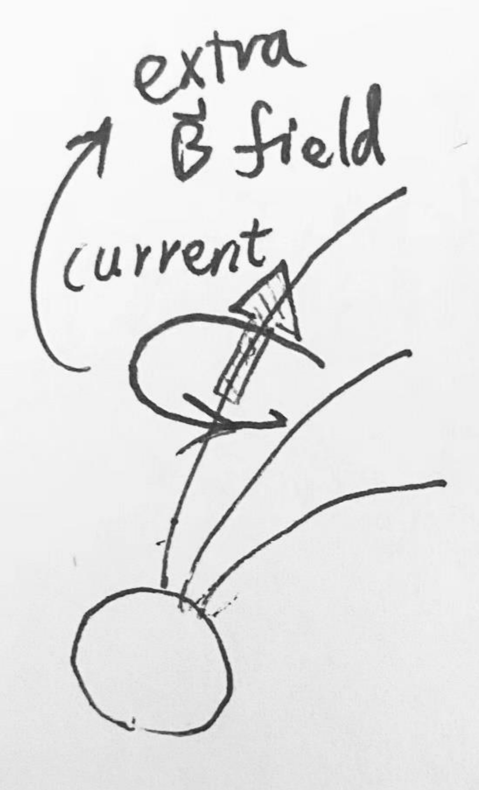
could vary on pulse phases.

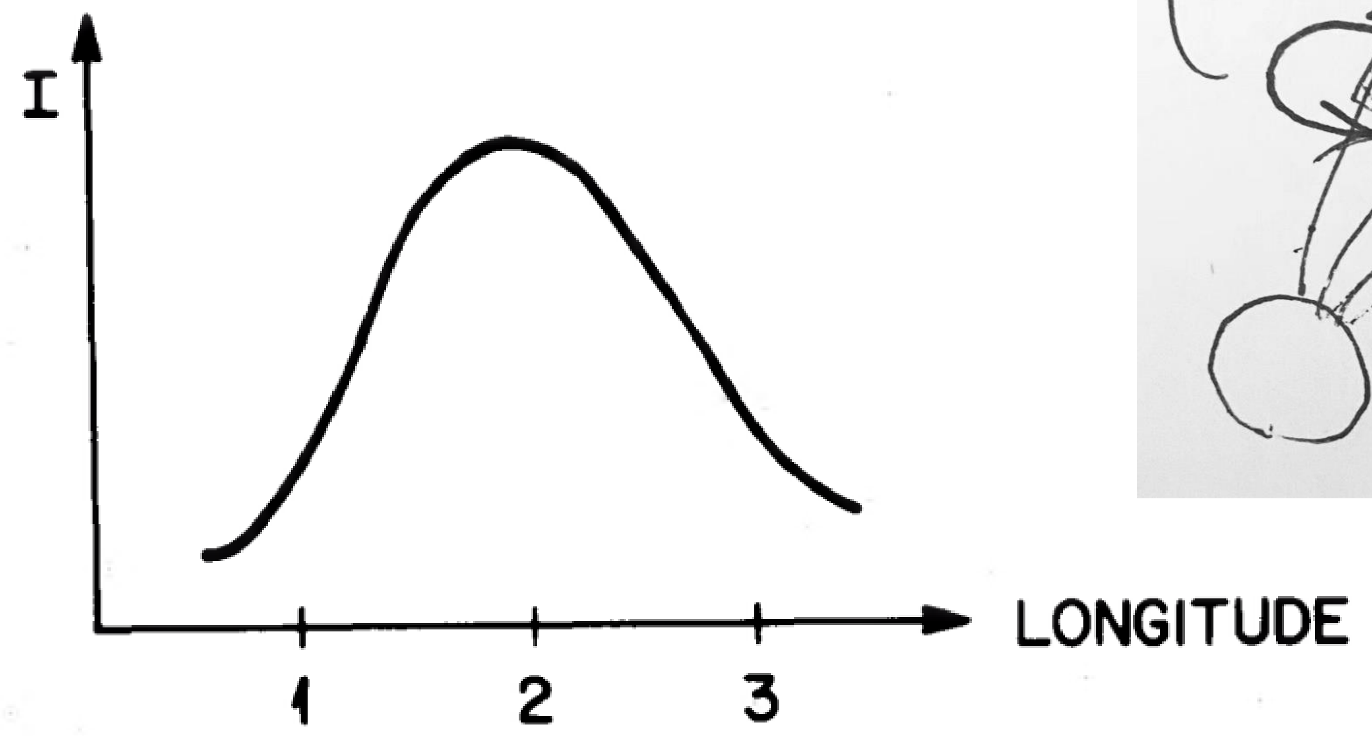
➔proportions of O & E mode could vary, too.

An example: $\omega_p{}^2 = 4\pi\bar{n}e^2/m$ changes because of particle density variations.



IV. Arise of Circular polarization



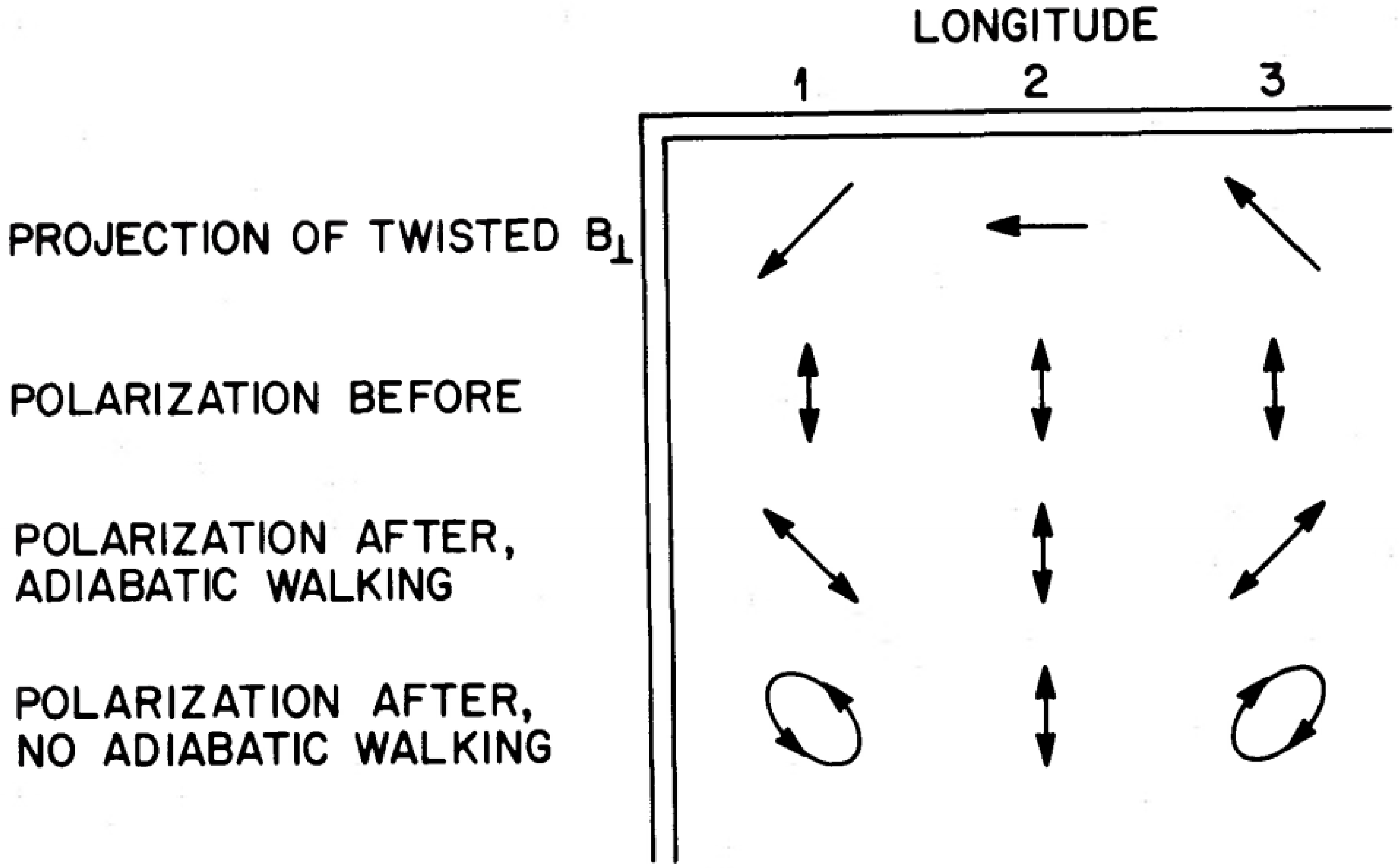


Firstly, recall: what makes linear polarization?

(1) Adiabatic walking
(2) Strong magnetic field $\omega \ll \gamma_w eB/mc$
(3) Electron/positron symmetry [not all need…?]

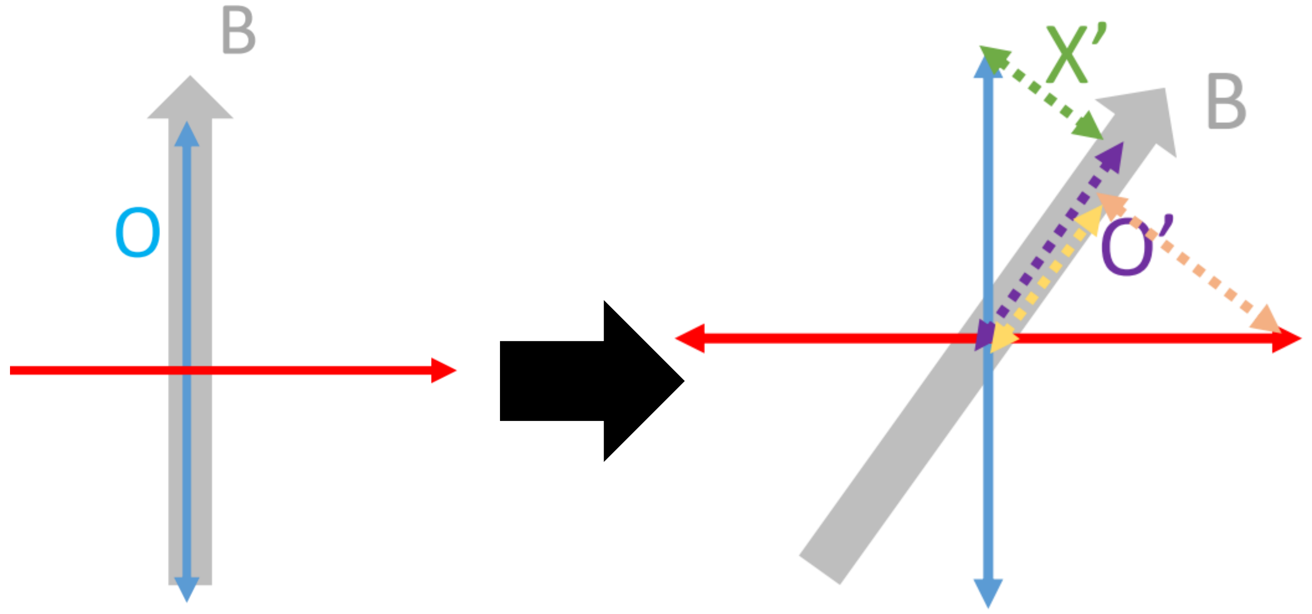
Case 1: (1) fails (within light cylinder) while (2) & (3) hold.

At r_0 $r_E < r_0 < r_{lc}$



Outer magnetosphere: B⊥ arises because of (rotation) (outward currents)…

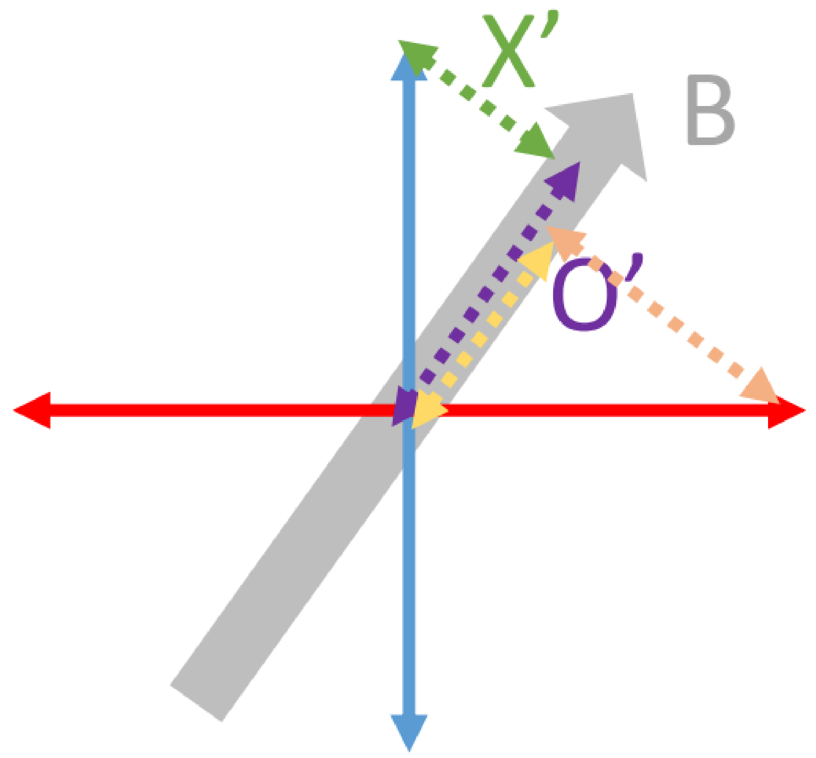
Circular polarization arises.



could cause circular polarization reversal

Firstly, recall: what makes linear polarization?

(1) Adiabatic walking
(2) Strong magnetic field $\omega \ll \gamma_w eB/mc$
(3) Electron/positron symmetry



Case 1: (1) fails (within light cylinder) while (2) & (3) hold.

At r_0 $r_E < r_0 < r_{lc}$

Estimate r_0: break of $\lambda\kappa \ll \Delta n$ $\left|\lambda \frac{\partial}{\partial s}\phi\right| \ll |\Delta n|$

$$\kappa \sim r_{lc}^{-1}$$

$$\Delta n \sim \frac{2}{\gamma_w^4\theta^2}\left(\frac{r_E}{r}\right)^3$$

$$\omega \sim \gamma_w^3 c(r_E r_{lc})^{-1/2}$$

$$\theta \sim r/r_{lc}$$

(Abberation)

$$r_0 \sim \gamma_w^{-1/5}(r_{lc}r_E)^{1/2}$$
$$\sim w r_{lc}\gamma_w^{-1/5},$$

w: width of pulse ~ 0.1

➔ r_0 within light cylinder.

Beyond r_0, birefringence introduce phase difference between new O & E modes.

$$\Delta\tau \equiv \int_{r_0}^{r_{lc}} \Delta n \frac{\omega}{c}\hat{k}\cdot dr \sim 2\int_{r_0}^{r_{lc}} \frac{\omega dr}{\theta^2\gamma_w^4 c}\left(\frac{r_E}{r}\right)^3$$
$$\sim \frac{w^3}{\gamma_w^{3/5}}\left\langle\frac{1}{\theta^2}\right\rangle$$

May not be large enough…

Firstly, recall: what makes linear polarization?

(1) Adiabatic walking
(2) Strong magnetic field $\omega \ll \gamma_w eB/mc$
(3) Electron/positron symmetry

Case 2: (2)(3) fails (within light cylinder) before (1) fails.

Electrons/positrons have different γs.

(1) holds: $\lambda\kappa \ll \frac{2}{\gamma_w \gamma_<^3 \theta^2}\left(\frac{r_E}{r}\right)^3$ $\quad \gamma_< < \gamma_w$

(2) holds (quantitatively from Melrose & Stoneham 1977):

$$1 \ll \frac{eB}{mc\omega\theta^2\gamma_<^3}$$

(2) fails before (1): not difficult within light cylinder (even near emission).

$$\kappa < \frac{2 r_E^3 \omega^2 m}{\gamma_w e B_s R^3} \qquad \kappa < 10^{-8}\ \mathrm{cm}^{-1}$$

$$\kappa \sim (r r_{lc})^{-1/2}$$

Larger Δn, larger Δτ

Normal modes are elliptically polarized.

Circular polarization depends on $\boldsymbol{B}\cdot\boldsymbol{\Omega}$

Inner magnetosphere: Adiabatic walking
➔Polarizations follow magnetic field

Incoherent mixing & competition between $\parallel$ & $\perp$
➔OPM transitions

Adiabatic walking fails
Asymmetry between e-+ & B diminishes
➔ Circular polarization

Thank you for your attention ☺