On the origin of orthogonal polarization modes in pulsar radio emission

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A&A, 2001

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(Shunshun Cao)
2023.11

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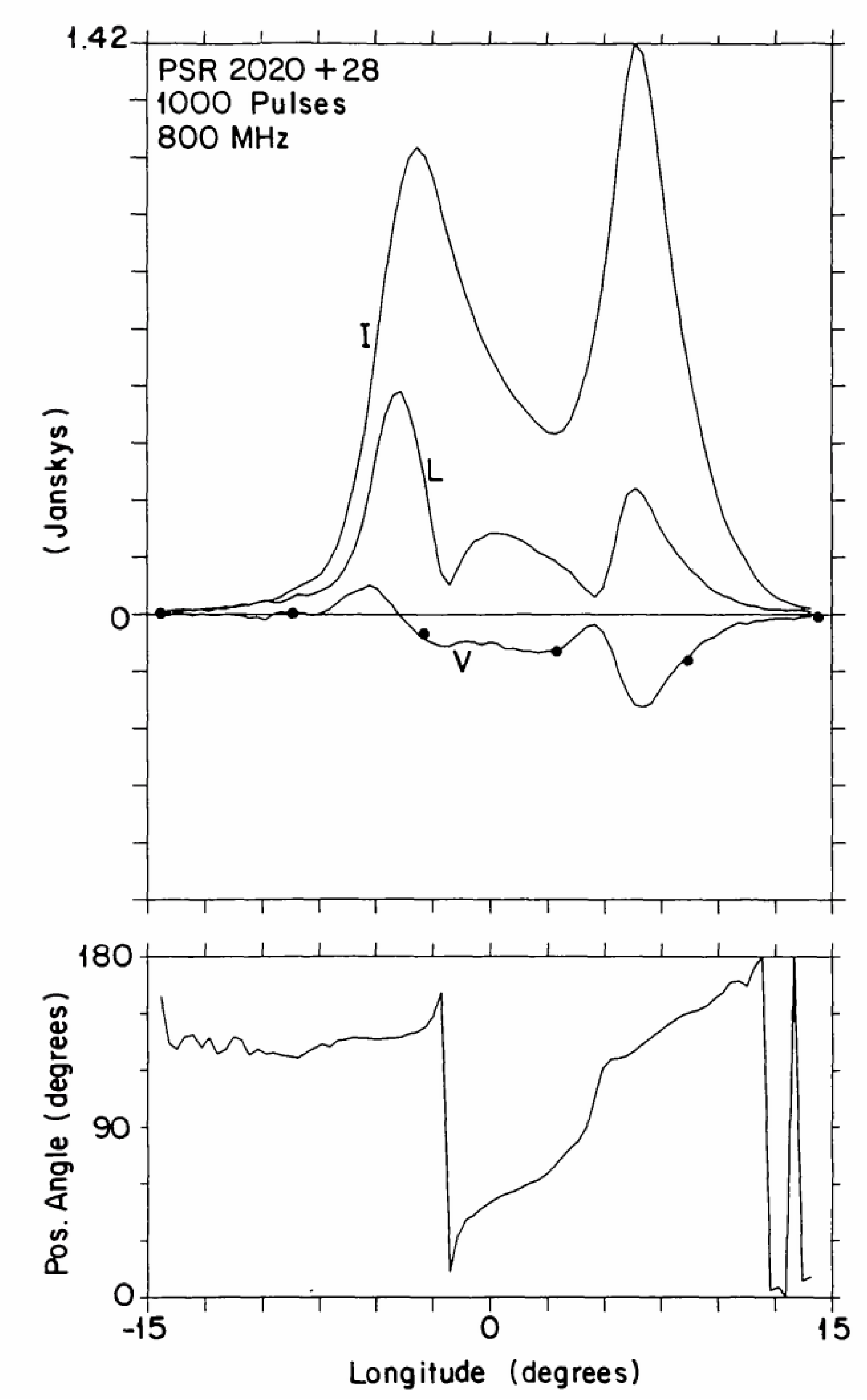
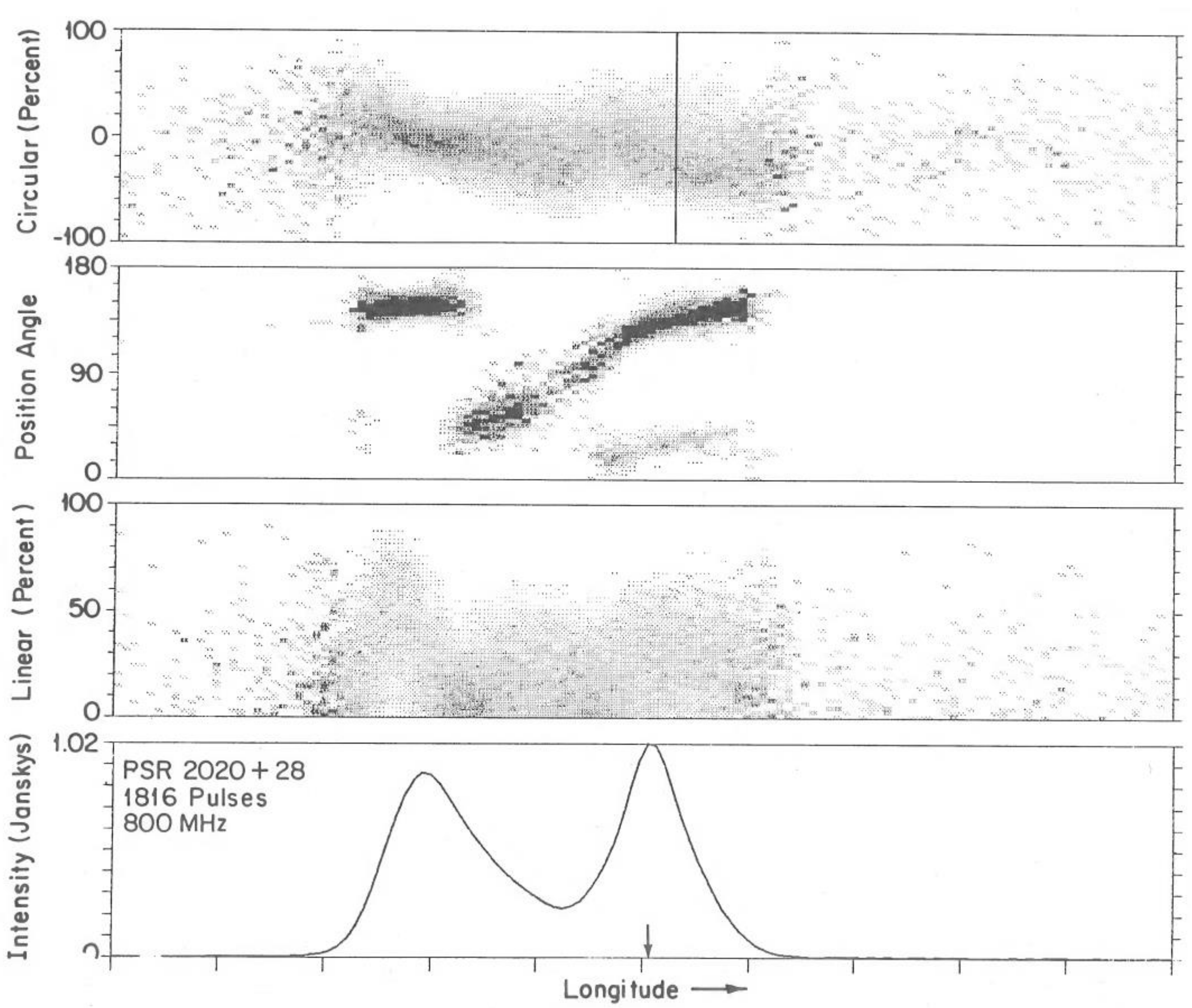
Contents

I. Introduction and Basic Picture

Orthogonal polarization modes (OPMs):

Phenomena shown in both integrated profiles and single pulses for pulsars.

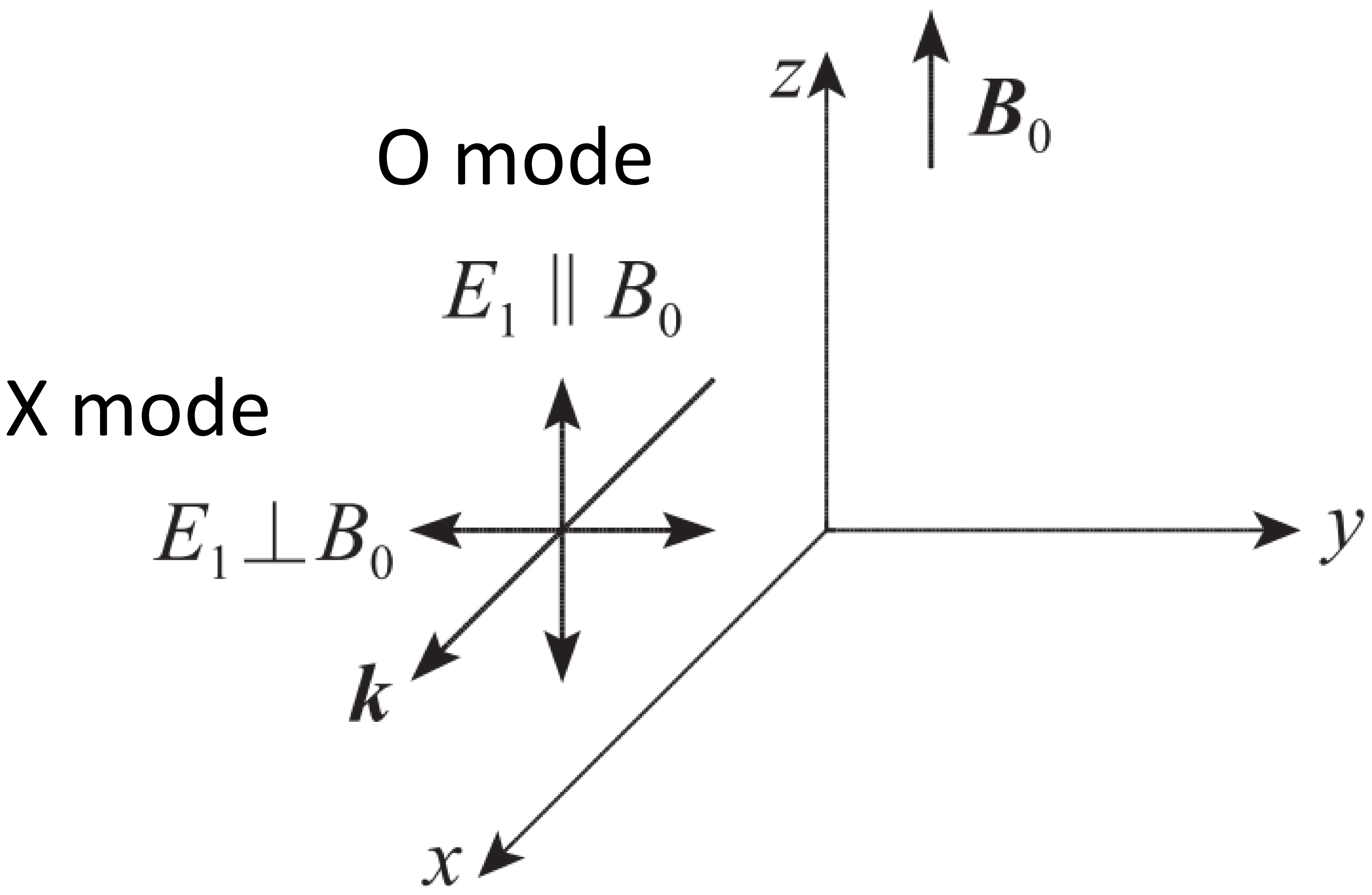




Stinebring et al. 1984

Physically: OPM ←→ Ordinary & Extraordinary wave modes in plasma.

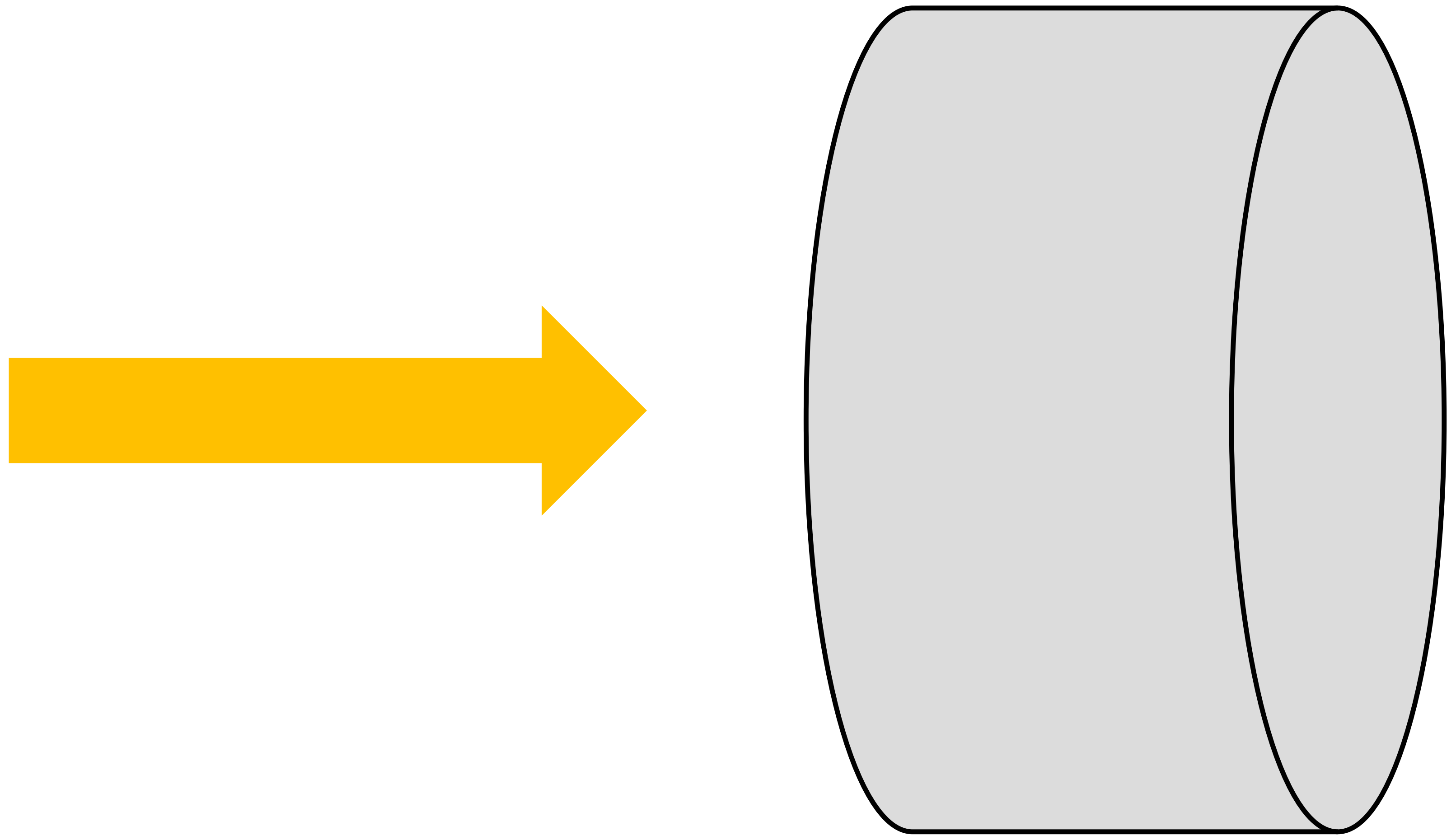
➔➔➔ How OPMs form? ➔➔➔ Dig into magnetosphere?

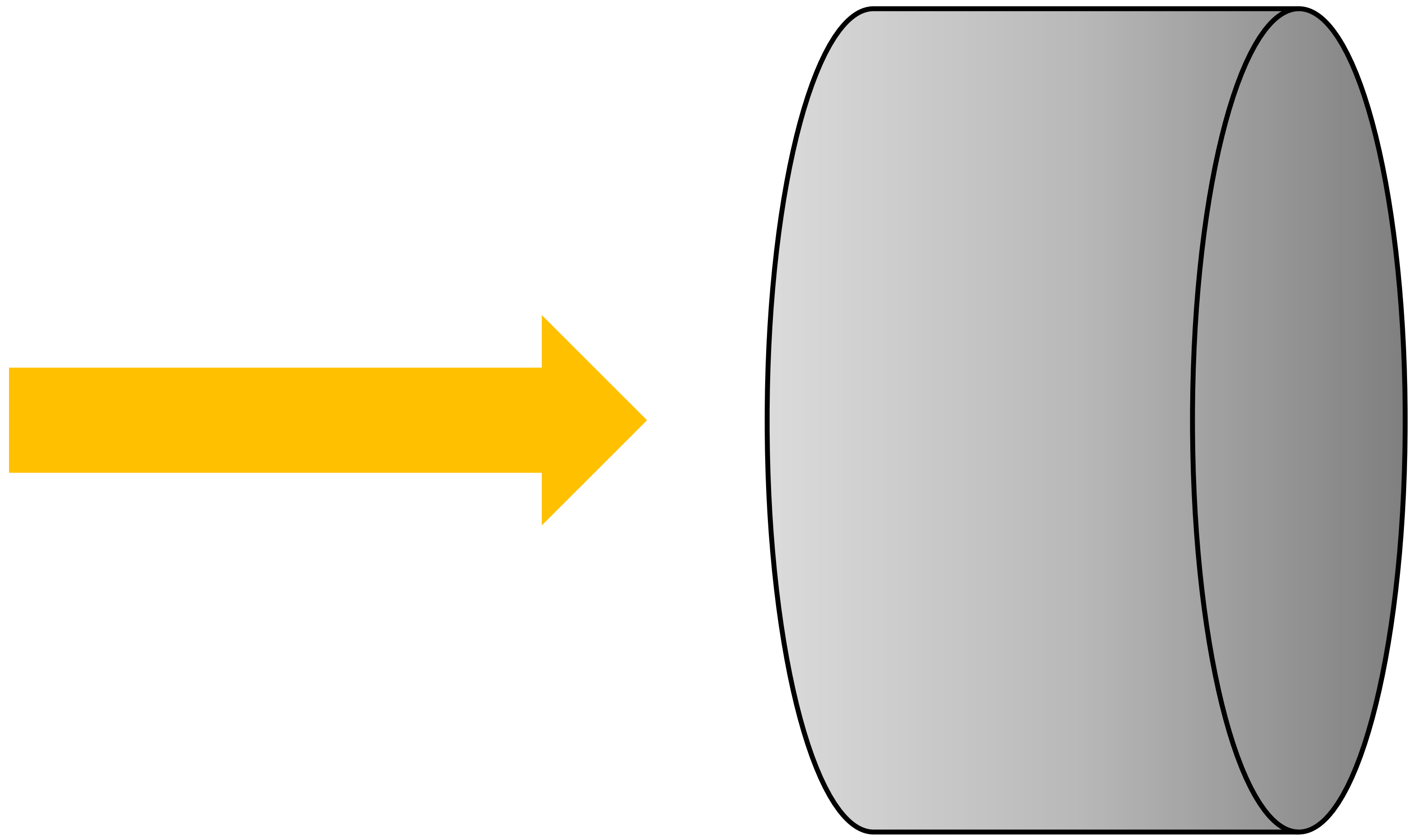


From F. F. Chen *Introduction to Plasma Physics*

Pulsar magnetosphere: inhomogeneous relativistic magnetized plasma.

Inhomogeneous: introduce geometrical optics approximation





$$\boldsymbol{E} \propto \exp(i\boldsymbol{k}_j\boldsymbol{r} - i\omega t)$$

$$\boldsymbol{E} = \boldsymbol{E}_a(\boldsymbol{r}) \exp\left(i \int \boldsymbol{k}_j d\boldsymbol{r} - i\omega t\right)$$

What we need for approximation: no violate changes in plasma physical condition

$$\frac{d\epsilon}{dz}\frac{\lambda}{2\pi} \ll \epsilon \qquad \frac{\omega}{c} n_j \Lambda \gg 1$$

From V. V. Zheleznyakov
Radiation in Astrophysical Plasma

When geometrical optics approximation holds:

$$\boldsymbol{E} \propto \exp(i\boldsymbol{k}_j\boldsymbol{r} - i\omega t)$$

j = 1,2 Different modes could have different n_j

In order to make sure wave modes propagate independently

➔ An additional need:

$$\frac{\omega}{c}|n_1 - n_2|\Lambda \gg 1$$

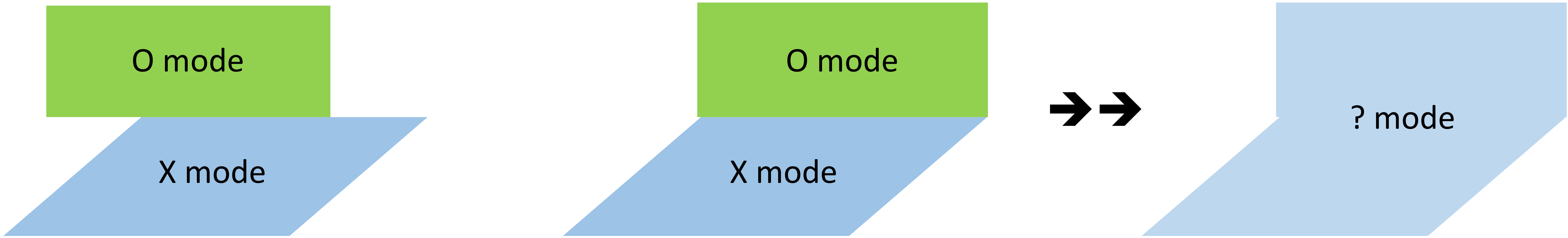
From V. V. Zheleznyakov
Radiation in Astrophysical Plasma

Λ is the characteristic scale for those parameters of a medium

Try to understand that: what if n1 (k1) is too close to n2 (k2)?

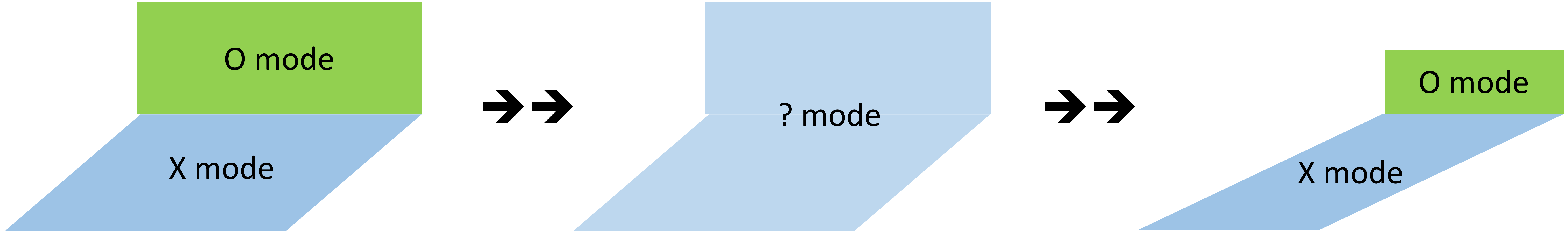
$$\frac{\omega}{c}|n_1 - n_2|\Lambda \gg 1$$

Can't tell between mode 1 & mode 2 (O mode and X mode).



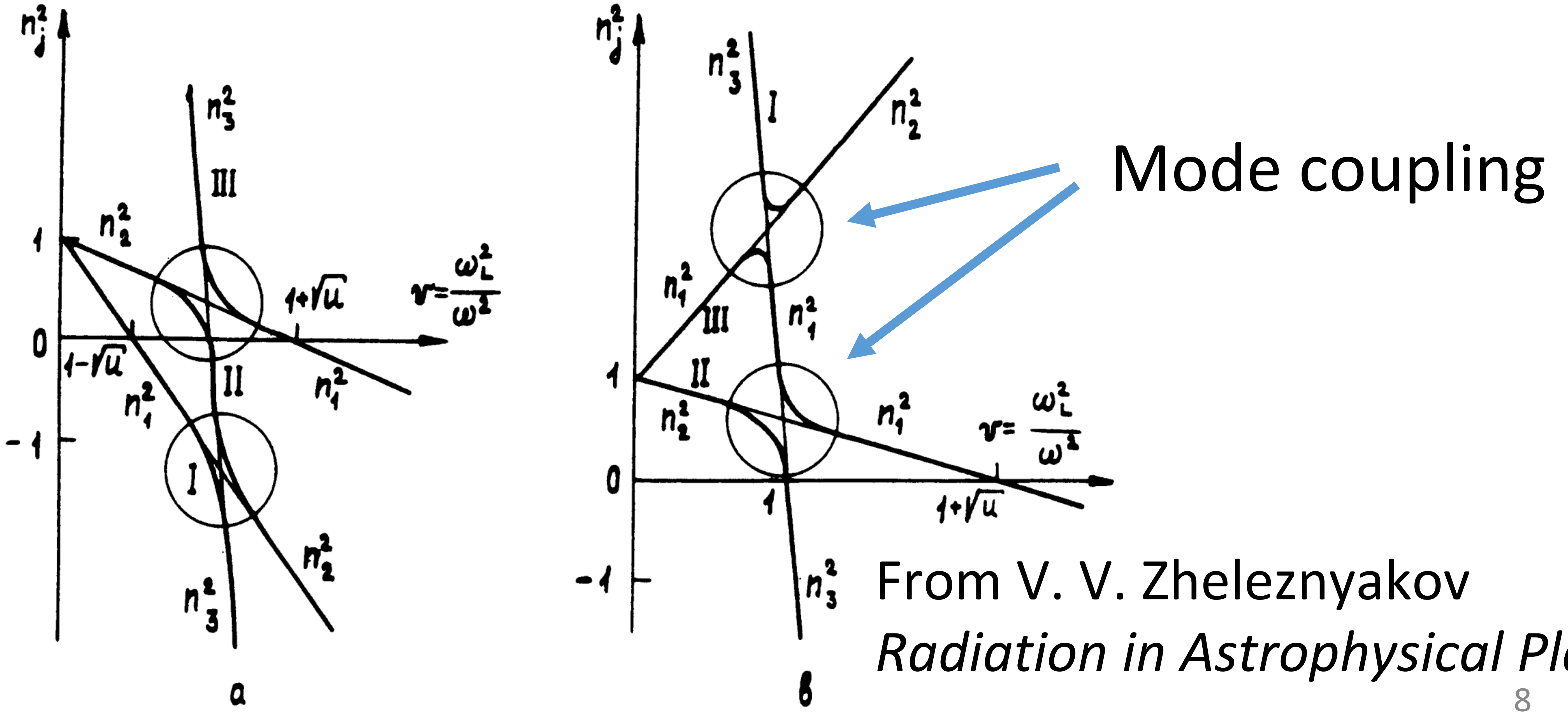
n1 > n2 n1 ≈ n2

Try to understand that: what if n1 (k1) is too close to n2 (k2)?



$n1 \approx n2$

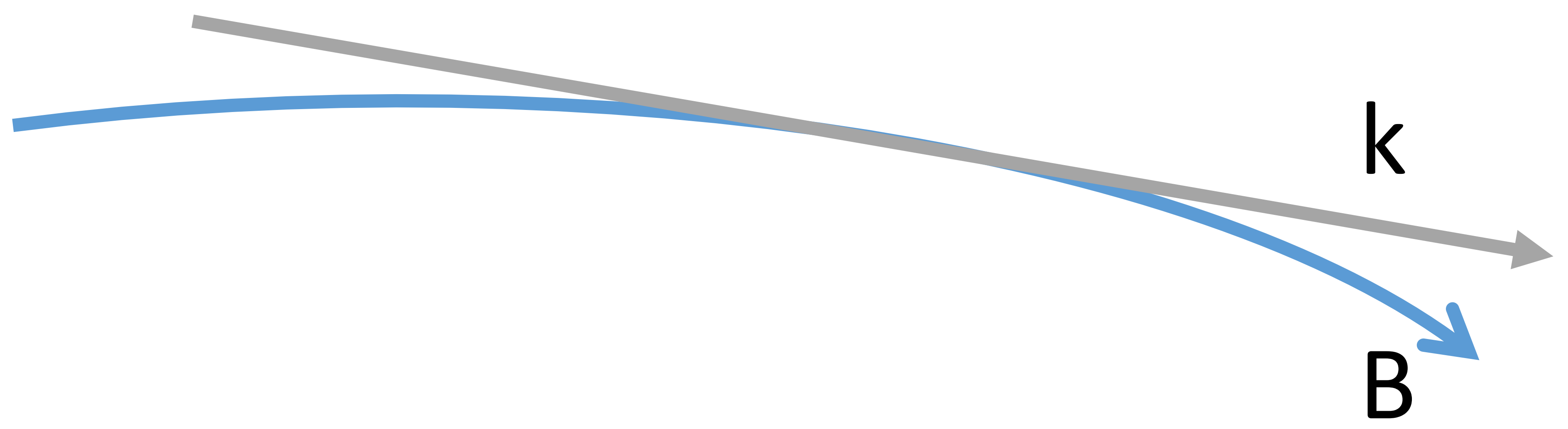
Conversion between O mode and X mode may happen.



From V. V. Zheleznyakov
Radiation in Astrophysical Plasma

When will O mode and X mode become indistinguishable?

One situation: quasi-longitudinal propagation (k // B).



By definition, when k//B, eigen wave modes are no longer purely linear.

➔➔ Search for k//B in pulsar magnetosphere?

➔➔➔ With wave **refraction**.

II. Equations and Results

(1) Propagation equations

Basic Eqs:

Maxwell's equations:

$$\nabla \times \boldsymbol{B} = -\frac{i\omega}{c}\boldsymbol{E} + \frac{4\pi}{c}\sum_{\alpha}\boldsymbol{j}_{\alpha},$$

$$\nabla \times \boldsymbol{E} = \frac{i\omega}{c}\boldsymbol{B},$$

Charge conservation: $-i\omega q_{\alpha} n_{\alpha} + \mathrm{div}\boldsymbol{j}_{\alpha} = 0$

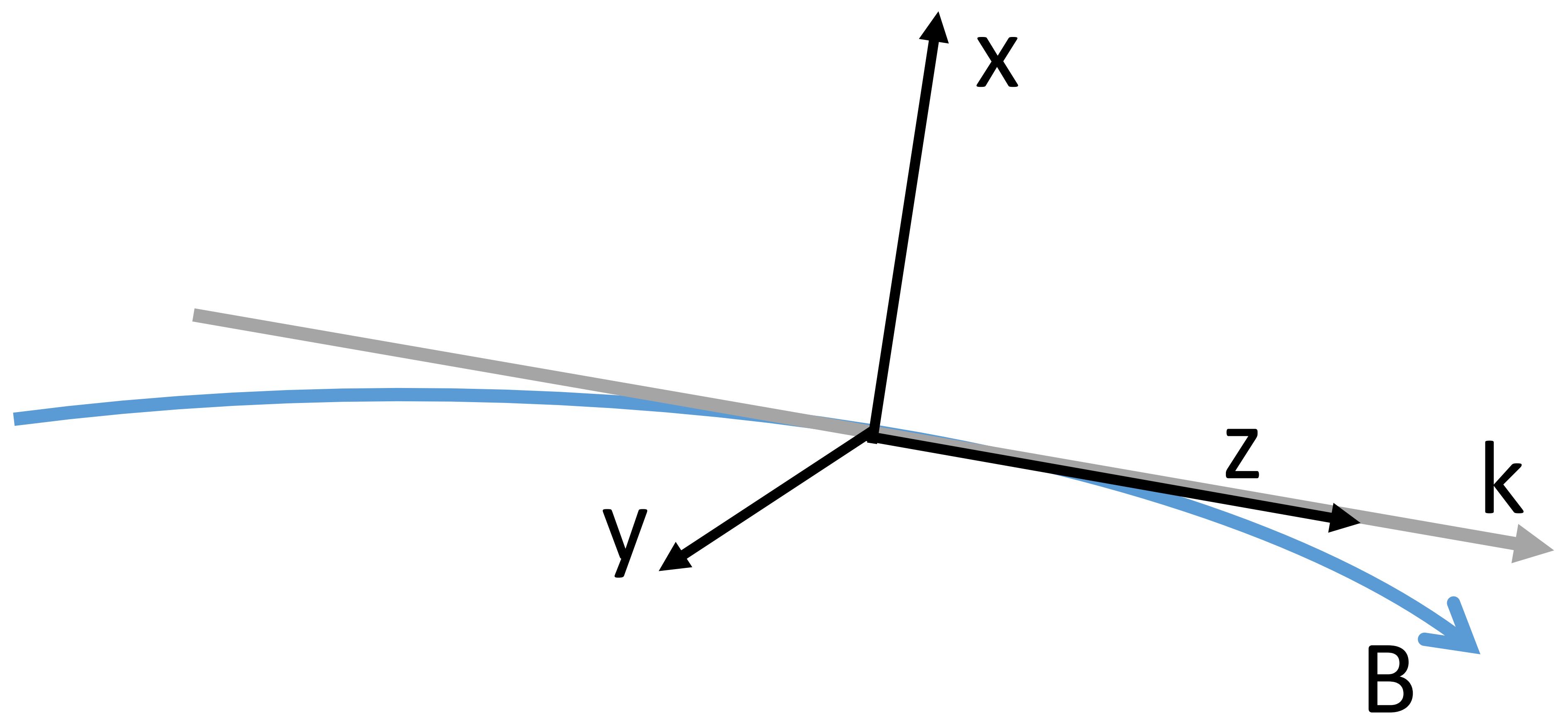
Current density: $\boldsymbol{j}_{\alpha} \equiv q_{\alpha}[n_{\alpha}\boldsymbol{v}_{0\alpha} + n_{0\alpha}\boldsymbol{v}_{\alpha}]$

Equation of motion: $\frac{\mathrm{d}\boldsymbol{p}_{\alpha}}{\mathrm{d}t} = q_{\alpha}\left(\boldsymbol{E} + \frac{\boldsymbol{v}_{\alpha} \times \boldsymbol{B}_0}{c} + \frac{\boldsymbol{v}_{0\alpha} \times \boldsymbol{B}}{c}\right)$

$$\frac{\mathrm{d}\boldsymbol{p}_{\parallel}}{\mathrm{d}t} = m\gamma^3\frac{\mathrm{d}\boldsymbol{v}_{\parallel}}{\mathrm{d}t}$$

$$\frac{\mathrm{d}\boldsymbol{p}_{\perp}}{\mathrm{d}t} = m\gamma\frac{\mathrm{d}\boldsymbol{v}_{\perp}}{\mathrm{d}t};$$

Coordinates:



$$\omega_{\mathrm{H}} \equiv \frac{eB_0}{mc} \qquad \omega' \equiv \gamma_\alpha \omega (1 - \beta_{0\alpha} b_z)$$

$$\boldsymbol{B}_0 = B_0 \boldsymbol{b}$$

$$\begin{aligned}
&-i\omega m \gamma_\alpha^3 (1 - \beta_{0\alpha} b_z) \left[v_{x\alpha} b_x + v_{y\alpha} b_y + v_{z\alpha} b_z \right] = \\
&q_\alpha (E_x b_x + E_y b_y + E_z b_z), \\
&-i\omega m \gamma_\alpha (1 - \beta_{0\alpha} b_z) \left[v_{y\alpha} b_x - v_{x\alpha} b_y \right] = \\
&q_\alpha (E_y b_x - E_x b_y)(1 - \beta_{0\alpha} b_z) \\
&+ q_\alpha \frac{B_0}{c} \left[v_{z\alpha} (b_x^2 + b_y^2) - v_{x\alpha} b_x b_z - v_{y\alpha} b_y b_z \right], \\
&i\omega m \gamma_\alpha (1 - \beta_{0\alpha} b_z) \left[v_{z\alpha} (b_x^2 + b_y^2) - v_{x\alpha} b_x b_z - v_{y\alpha} b_y b_z \right] = \\
&q_\alpha (E_x b_x + E_y b_y)(b_z - \beta_{0\alpha}) - q_\alpha E_z (b_x^2 + b_y^2) \\
&+ q_\alpha \frac{B_0}{c} \left[v_{y\alpha} b_x - v_{x\alpha} b_y \right], \qquad (7)
\end{aligned}$$

perturbation velocities

➔➔➔

$$\begin{aligned}
v_{x\alpha} &= \frac{q_\alpha^2 B_0 \left[E_y b_z - \beta_{0\alpha} E_x b_x b_y - \beta_{0\alpha} E_y (b_y^2 + b_z^2) - E_z b_y \right]}{m^2 c (\omega_{\mathrm{H}}^2 - \omega'^2)} \\
&+ \frac{i q_\alpha \omega'}{m(\omega_{\mathrm{H}}^2 - \omega'^2)} \left[E_y b_x b_y + E_z b_x b_z + \beta_{0\alpha} E_x b_z \right. \\
&\left. - E_x (b_y^2 + b_z^2) \right] + \frac{i q_\alpha b_x [E_x b_x + E_y b_y + E_z b_z]}{m \gamma_\alpha^3 \omega (1 - \beta_{0\alpha} b_z)}, \\
v_{y\alpha} &= \frac{q_\alpha^2 B_0 \left[E_x (\beta_{0\alpha} (b_x^2 + b_z^2) - b_z) + \beta_{0\alpha} E_y b_x b_y + b_x E_z \right]}{m^2 c (\omega_{\mathrm{H}}^2 - \omega'^2)} \\
&+ \frac{i q_\alpha \omega'}{m(\omega_{\mathrm{H}}^2 - \omega'^2)} \left[E_x b_x b_y + E_y (\beta_{0\alpha} b_z - (b_x^2 + b_z^2)) \right. \\
&\left. + b_y b_z E_z \right] + \frac{i q_\alpha b_y [E_x b_x + E_y b_y + E_z b_z]}{m \gamma_\alpha^3 \omega (1 - \beta_{0\alpha} b_z)}, \qquad (8) \\
v_{z\alpha} &= \frac{i q_\alpha b_z [E_x b_x + E_y b_y + E_z b_z]}{m \gamma_\alpha^3 \omega (1 - \beta_{0\alpha} b_z)} \\
&+ \frac{i q_\alpha \omega'}{m(\omega_{\mathrm{H}}^2 - \omega'^2)} \left[(E_x b_x + E_y b_y)(b_z - \beta_{0\alpha}) \right. \\
&\left. - E_z (b_x^2 + b_y^2) \right] - \frac{q_\alpha^2 B_0 (E_y b_x - E_x b_y)(1 - \beta_{0\alpha} b_z)}{m^2 c (\omega_{\mathrm{H}}^2 - \omega'^2)},
\end{aligned}$$

Maxwell's equations:

$$\nabla \times \boldsymbol{B} = -\frac{i\omega}{c}\boldsymbol{E} + \frac{4\pi}{c}\sum_{\alpha}\boldsymbol{j}_{\alpha},$$

$$\nabla \times \boldsymbol{E} = \frac{i\omega}{c}\boldsymbol{B},$$

Charge conservation:

$$-i\omega q_{\alpha} n_{\alpha} + \mathrm{div}\boldsymbol{j}_{\alpha} = 0$$

$$n_{\alpha} = \frac{n_{0\alpha} v_{z\alpha}/c}{1 - \beta_{0\alpha} b_z}$$

$$\frac{\mathrm{d}E_x}{\mathrm{d}z} + \frac{2\pi}{c}\sum_{\alpha}\frac{q_{\alpha} n_{0\alpha}}{1-\beta_{0\alpha} b_z}[v_{x\alpha}(1-\beta_{0\alpha} b_z) - v_{z\alpha}\beta_{0\alpha} b_x] = 0, (10)$$

$$\frac{\mathrm{d}E_y}{\mathrm{d}z} + \frac{2\pi}{c}\sum_{\alpha}\frac{q_{\alpha} n_{0\alpha}}{1-\beta_{0\alpha} b_z}[v_{y\alpha}(1-\beta_{0\alpha} b_z) + v_{z\alpha}\beta_{0\alpha} b_y] = 0,$$

$$E_z + \frac{4\pi i}{\omega}\sum_{\alpha}\frac{q_{\alpha} n_{0\alpha} v_{z\alpha}}{1 - \beta_{0\alpha} b_z} = 0.$$

$$\frac{\mathrm{d}E_x}{\mathrm{d}z}+\frac{2\pi}{c}\sum_\alpha \frac{q_\alpha n_{0\alpha}}{1-\beta_{0\alpha}b_z}[v_{x\alpha}(1-\beta_{0\alpha}b_z)-v_{z\alpha}\beta_{0\alpha}b_x]=0, \quad (10)$$

$$\frac{\mathrm{d}E_y}{\mathrm{d}z}+\frac{2\pi}{c}\sum_\alpha \frac{q_\alpha n_{0\alpha}}{1-\beta_{0\alpha}b_z}[v_{y\alpha}(1-\beta_{0\alpha}b_z)+v_{z\alpha}\beta_{0\alpha}b_y]=0,$$

$$E_z + \frac{4\pi i}{\omega}\sum_\alpha \frac{q_\alpha n_{0\alpha} v_{z\alpha}}{1-\beta_{0\alpha}b_z}=0.$$

$$\begin{aligned} v_{x\alpha} &= \frac{q_\alpha^2 B_0\left[E_y b_z-\beta_{0\alpha}E_x b_x b_y-\beta_{0\alpha}E_y(b_y^2+b_z^2)-E_z b_y\right]}{m^2 c(\omega_{\mathrm{H}}^2-\omega'^2)} \\ &+\frac{i q_\alpha \omega'}{m(\omega_{\mathrm{H}}^2-\omega'^2)}\left[E_y b_x b_y+E_z b_x b_z+\beta_{0\alpha}E_x b_z\right. \\ &\left.-E_x(b_y^2+b_z^2)\right]+\frac{i q_\alpha b_x[E_x b_x+E_y b_y+E_z b_z]}{m\gamma_\alpha^3\omega(1-\beta_{0\alpha}b_z)}, \\ v_{y\alpha} &= \frac{q_\alpha^2 B_0\left[E_x(\beta_{0\alpha}(b_x^2+b_z^2)-b_z)+\beta_{0\alpha}E_y b_x b_y+b_x E_z\right]}{m^2 c(\omega_{\mathrm{H}}^2-\omega'^2)} \\ &+\frac{i q_\alpha \omega'}{m(\omega_{\mathrm{H}}^2-\omega'^2)}\left[E_x b_x b_y+E_y(\beta_{0\alpha}b_z-(b_x^2+b_z^2))\right. \\ &\left.+b_y b_z E_z\right]+\frac{i q_\alpha b_y[E_x b_x+E_y b_y+E_z b_z]}{m\gamma_\alpha^3\omega(1-\beta_{0\alpha}b_z)}, \qquad (8) \\ v_{z\alpha} &= \frac{i q_\alpha b_z[E_x b_x+E_y b_y+E_z b_z]}{m\gamma_\alpha^3\omega(1-\beta_{0\alpha}b_z)} \\ &+\frac{i q_\alpha \omega'}{m(\omega_{\mathrm{H}}^2-\omega'^2)}\left[(E_x b_x+E_y b_y)(b_z-\beta_{0\alpha})\right. \\ &\left.-E_z(b_x^2+b_y^2)\right]-\frac{q_\alpha^2 B_0(E_y b_x-E_x b_y)(1-\beta_{0\alpha}b_z)}{m^2 c(\omega_{\mathrm{H}}^2-\omega'^2)}, \end{aligned}$$

➡

$$\begin{aligned} E_z &= \sum_\alpha \frac{\omega_{\mathrm{p}\alpha}^2 b_z}{\gamma_\alpha \omega'^2}(E_x b_x+E_y b_y+E_z b_z) \\ &+\sum_\alpha \frac{\omega_{\mathrm{p}\alpha}^2\gamma_\alpha}{\omega_{\mathrm{H}}^2-\omega'^2}[(E_x b_x+E_y b_y)(b_z-\beta_{0\alpha})-E_z(b_x^2+b_y^2)] \\ &+\sum_\alpha \frac{i(q_\alpha/e)\omega_{\mathrm{H}}\omega_{\mathrm{p}\alpha}^2\gamma_\alpha}{\omega'(\omega_{\mathrm{H}}^2-\omega'^2)}(E_y b_x-E_x b_y)(1-\beta_{0\alpha}b_z), \end{aligned} \qquad (11)$$

High frequency & Strong magnetic field

$$\omega_{\mathrm{p}\alpha} \equiv \sqrt{\frac{4\pi e^2 n_{0\alpha}}{m}} \qquad \frac{\omega'}{\omega_{\mathrm{H}}} \ll 1$$

$$E_z \ll E_x, E_y$$

Quasi-Transverse

$$\frac{dE_x}{dz}+\frac{2\pi}{c}\sum_\alpha \frac{q_\alpha n_{0\alpha}}{1-\beta_{0\alpha}b_z}[v_{x\alpha}(1-\beta_{0\alpha}b_z)-v_{z\alpha}\beta_{0\alpha}b_x]=0, \quad (10)$$

$$\frac{dE_y}{dz}+\frac{2\pi}{c}\sum_\alpha \frac{q_\alpha n_{0\alpha}}{1-\beta_{0\alpha}b_z}[v_{y\alpha}(1-\beta_{0\alpha}b_z)+v_{z\alpha}\beta_{0\alpha}b_y]=0,$$

$$E_z + \frac{4\pi i}{\omega}\sum_\alpha \frac{q_\alpha n_{0\alpha}v_{z\alpha}}{1-\beta_{0\alpha}b_z}=0.$$

$$\begin{aligned} v_{x\alpha} &= \frac{q_\alpha^2 B_0\left[E_y b_z - \beta_{0\alpha}E_x b_x b_y - \beta_{0\alpha}E_y(b_y^2+b_z^2) - E_z b_y\right]}{m^2 c(\omega_{\rm H}^2-\omega'^2)} \\ &+\frac{iq_\alpha\omega'}{m(\omega_{\rm H}^2-\omega'^2)}\left[E_y b_x b_y + E_z b_x b_z + \beta_{0\alpha}E_x b_z \right. \\ &\left. - E_x(b_y^2+b_z^2)\right] + \frac{iq_\alpha b_x[E_x b_x + E_y b_y + E_z b_z]}{m\gamma_\alpha^3\omega(1-\beta_{0\alpha}b_z)}, \\ v_{y\alpha} &= \frac{q_\alpha^2 B_0\left[E_x(\beta_{0\alpha}(b_x^2+b_z^2)-b_z)+\beta_{0\alpha}E_y b_x b_y + b_x E_z\right]}{m^2 c(\omega_{\rm H}^2-\omega'^2)} \\ &+\frac{iq_\alpha\omega'}{m(\omega_{\rm H}^2-\omega'^2)}\left[E_x b_x b_y + E_y(\beta_{0\alpha}b_z - (b_x^2+b_z^2)) \right. \\ &\left. + b_y b_z E_z\right] + \frac{iq_\alpha b_y[E_x b_x + E_y b_y + E_z b_z]}{m\gamma_\alpha^3\omega(1-\beta_{0\alpha}b_z)}, \qquad (8) \\ v_{z\alpha} &= \frac{iq_\alpha b_z[E_x b_x + E_y b_y + E_z b_z]}{m\gamma_\alpha^3\omega(1-\beta_{0\alpha}b_z)} \\ &+\frac{iq_\alpha\omega'}{m(\omega_{\rm H}^2-\omega'^2)}\left[(E_x b_x + E_y b_y)(b_z-\beta_{0\alpha}) \right. \\ &\left. - E_z(b_x^2+b_y^2)\right] - \frac{q_\alpha^2 B_0(E_y b_x - E_x b_y)(1-\beta_{0\alpha}b_z)}{m^2 c(\omega_{\rm H}^2-\omega'^2)}, \end{aligned}$$

$$\frac{dE_x}{dz} + \frac{i\omega}{2c}[Ab_x(E_x b_x + E_y b_y) - BE_x + iGE_y] = 0,$$

$$\frac{dE_y}{dz} + \frac{i\omega}{2c}[Ab_y(E_x b_x + E_y b_y) - BE_y - iGE_x] = 0, \quad (12)$$

where

$$A \equiv \sum_\alpha \frac{\omega_{{\rm p}\alpha}^2}{\gamma_\alpha\omega'^2}\frac{\omega_{\rm H}^2}{\omega_{\rm H}^2-\omega'^2};$$

$$B \equiv \sum_\alpha \frac{\omega_{{\rm p}\alpha}^2\gamma_\alpha(1-\beta_{0\alpha}b_z)^2}{\omega_{\rm H}^2-\omega'^2};$$

$$G \equiv \sum_\alpha \frac{i(q_\alpha/e)(\omega_{\rm H}/\omega)\omega_{{\rm p}\alpha}^2(\beta_{0\alpha}-b_z)}{\omega_{\rm H}^2-\omega'^2}.$$

Propagation equations

(2) Refraction (in pulsar magnetosphere)
(Barnard & Arons 1986...)

The effect of magnetospheric refraction on the morphology of pulsar profiles

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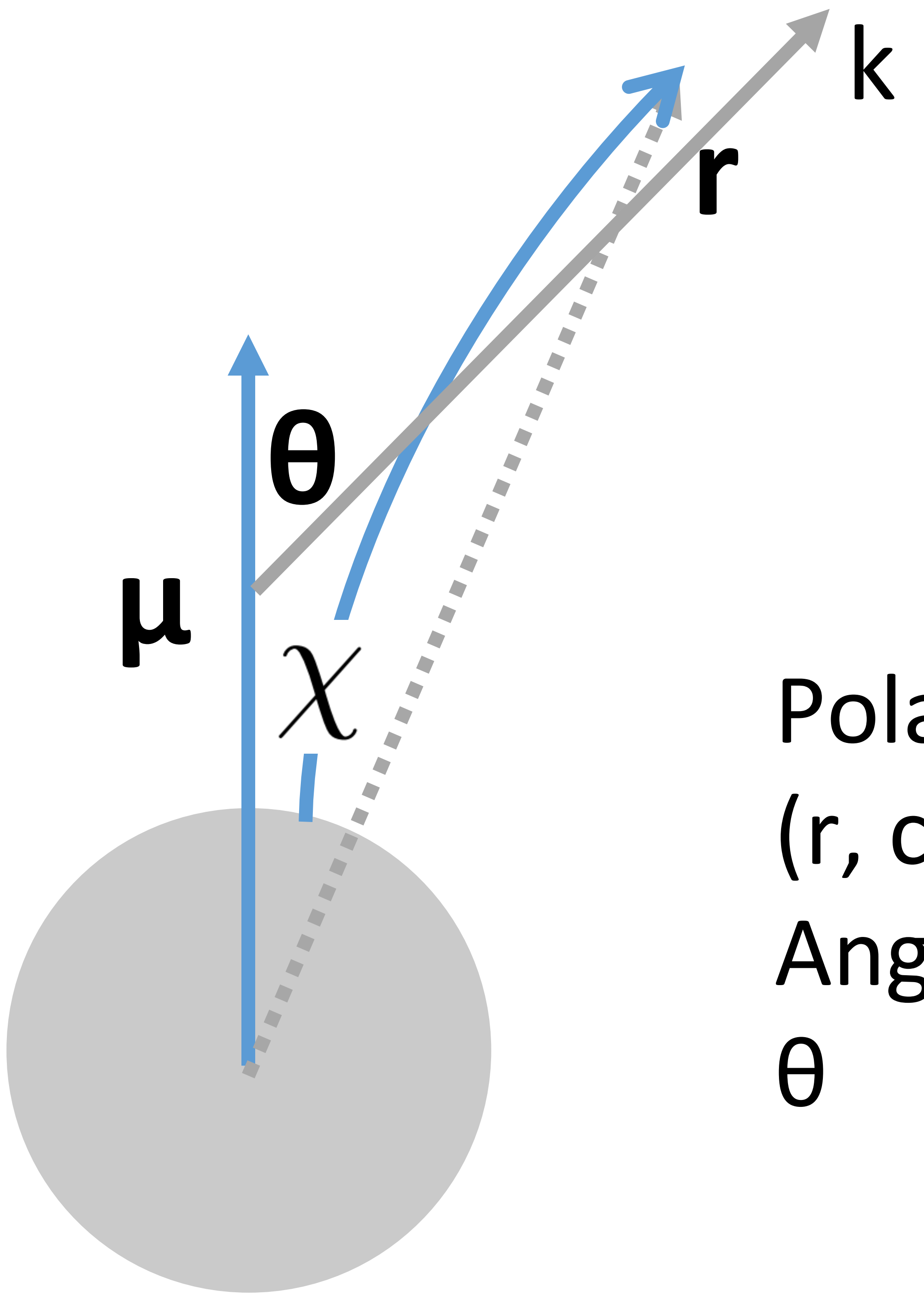
$$\left(1-n_{\|}^2\right)\left(1-\frac{\omega_p^2}{\omega^2\gamma^3(1-n_{\|}\beta)^2}\right)-n_{\perp}^2=0,$$

Plasma particles' distribution: hollow-cone-like

$$N=\begin{cases} N_0\left(\frac{r_0}{r}\right)^3\exp\left(-\varepsilon_1\frac{(\chi-\chi_c\sqrt{r/r_0})^2}{(\chi_c\sqrt{r/r_0})^2}\right), \\ \qquad |\chi|\le\chi_c\sqrt{r/r_0} \\ N_0\left(\frac{r_0}{r}\right)^3\exp\left(-\varepsilon_2\frac{(\chi-\chi_c\sqrt{r/r_0})^2}{(\chi_c\sqrt{r/r_0})^2}\right), \\ \qquad |\chi|\ge\chi_c\sqrt{r/r_0}, \end{cases}$$



Polar coordinate:
(r, chi)
Angle between k and μ:
θ

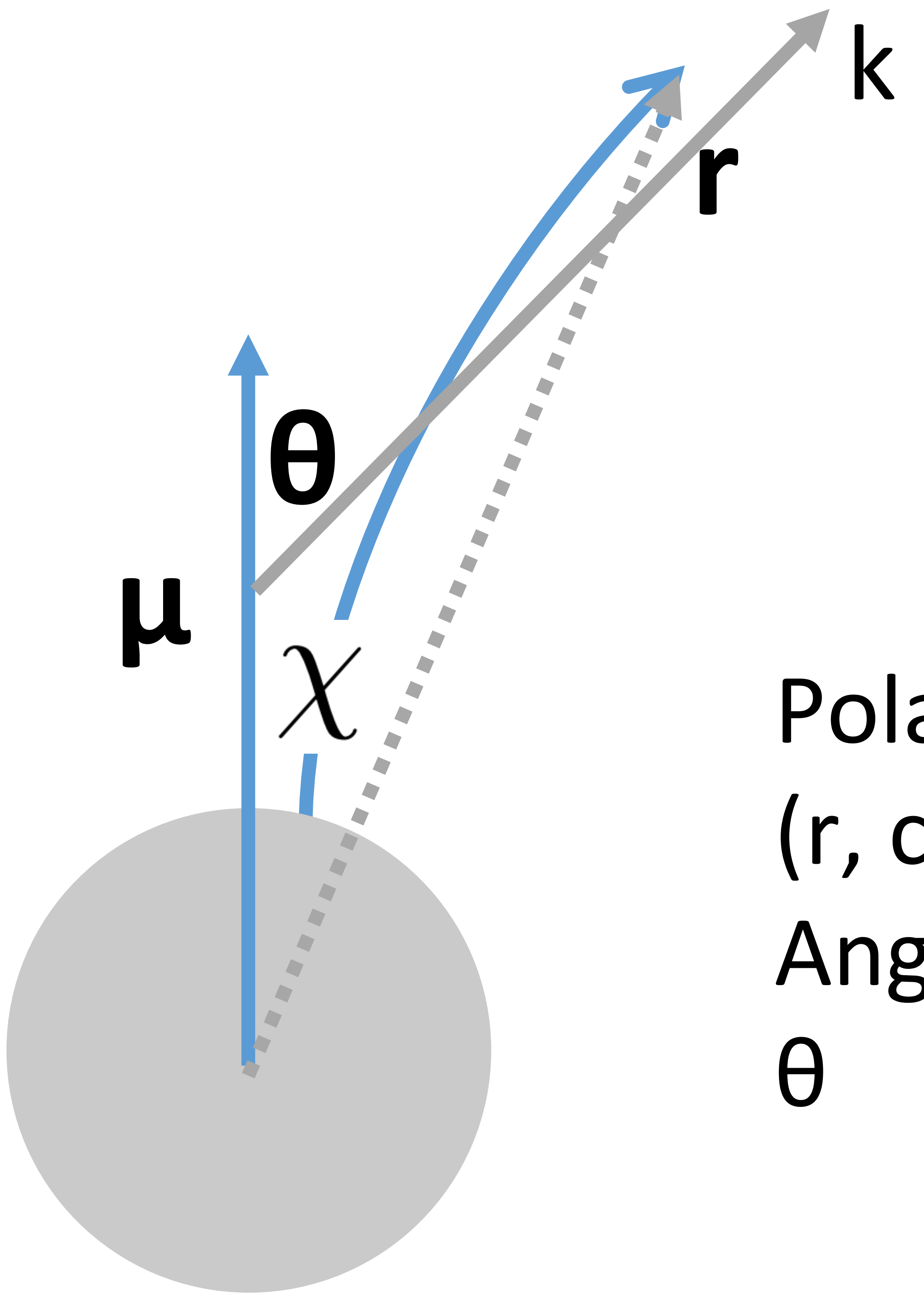
Introduce $\chi_{\rm f} \quad \theta_{\rm f} \quad z_{\rm f}$

Further than $\chi_{\rm f} \quad \theta_{\rm f} \quad z_{\rm f}$, particle density too low, magnetic field too small:

➔➔ Refraction could be ignored. The ray propagates in a straight line. (Angles are small)

$$\chi = \theta_{\rm f} - \frac{z_{\rm f}}{z}(\theta_{\rm f} - \chi_{\rm f})$$

$$b_x = \frac{3}{2}\chi - \theta_{\rm f} = \frac{\theta_{\rm f}}{2} - \frac{3z_{\rm f}}{2z}(\theta_{\rm f} - \chi_{\rm f})$$



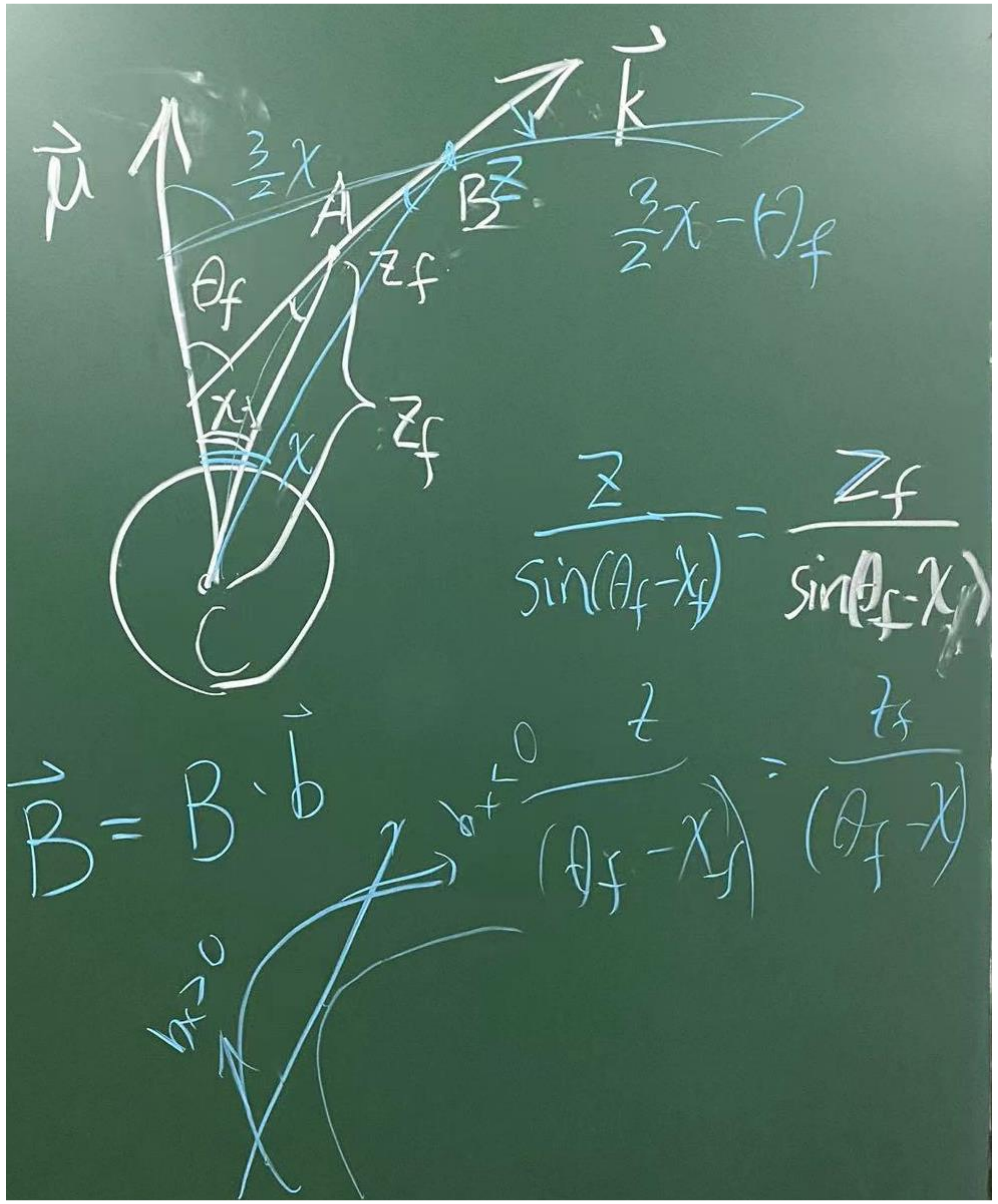
Polar coordinate:
(r, chi)
Angle between k and μ:
θ

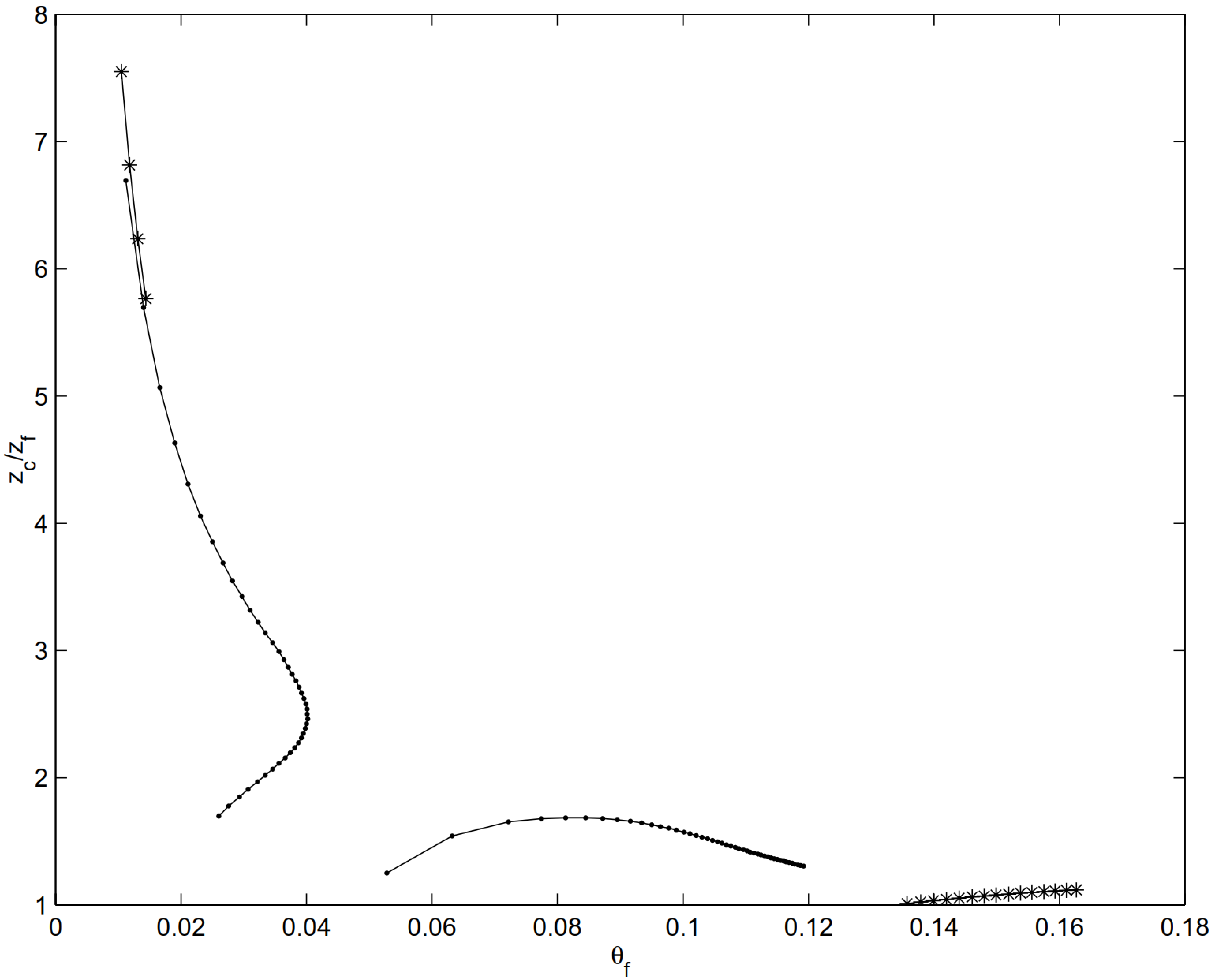
If $\theta_{\rm f} > \frac{3}{2}\chi_{\rm f}$

Then At $z > z_{\rm f}$

Zc

There bx changes sign. ➔➔➔ Longitudinal propagation exists.





8
7
6
5
4
3
2
1
z_c/z_f
0
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16
0.18
θ_f

(3) Linear conversion
(3.1) In the limit $B_0 \to \infty$ $\omega_H \to \infty$

$$\frac{\mathrm{d}E_x}{\mathrm{d}z} + \frac{i\omega}{2c}[Ab_x(E_x b_x + E_y b_y) - BE_x + iGE_y] = 0,$$

$$\frac{\mathrm{d}E_y}{\mathrm{d}z} + \frac{i\omega}{2c}[Ab_y(E_x b_x + E_y b_y) - BE_y - iGE_x] = 0, \quad (12)$$

where

$$A \equiv \sum_\alpha \frac{\omega_{\mathrm{p}\alpha}^2}{\gamma_\alpha \omega'^2} \frac{\omega_{\mathrm{H}}^2}{\omega_{\mathrm{H}}^2 - \omega'^2};$$

$$B \equiv \sum_\alpha \frac{\omega_{\mathrm{p}\alpha}^2 \gamma_\alpha (1 - \beta_{0\alpha} b_z)^2}{\omega_{\mathrm{H}}^2 - \omega'^2};$$

$$G \equiv \sum_\alpha \frac{i(q_\alpha/e)(\omega_{\mathrm{H}}/\omega)\omega_{\mathrm{p}\alpha}^2(\beta_{0\alpha} - b_z)}{\omega_{\mathrm{H}}^2 - \omega'^2}.$$

$\Rightarrow$

$$\frac{\mathrm{d}E_x}{\mathrm{d}z} + iRb_x(E_x b_x + E_y b_y) = 0,$$

$$\frac{\mathrm{d}E_y}{\mathrm{d}z} + iRb_y(E_x b_x + E_y b_y) = 0,$$

where

$$R \equiv \sum_\alpha \frac{\omega_{\mathrm{p}\alpha}^2}{2\omega c \gamma_\alpha^3 (1 - \beta_{0\alpha} b_z)^2}.$$

If adapting geometrical approximation: consider b_x, b_y changing very slowly with z.

$$\frac{\mathrm{d}E_x}{\mathrm{d}z} + iRb_x(E_x b_x + E_y b_y) = 0,$$
$$\frac{\mathrm{d}E_y}{\mathrm{d}z} + iRb_y(E_x b_x + E_y b_y) = 0,$$

where

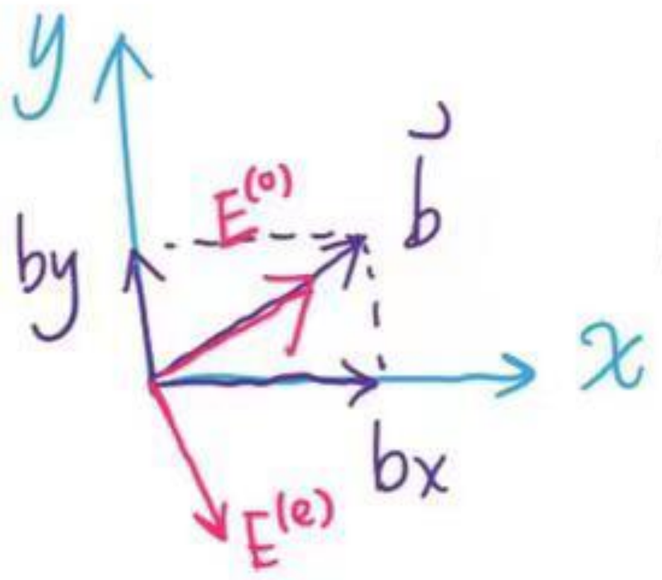
$$R \equiv \sum_\alpha \frac{\omega_{\mathrm{p}\alpha}^2}{2\omega c \gamma_\alpha^3 (1 - \beta_{0\alpha} b_z)^2}.$$

$$E_x^{(\mathrm{o})} = \frac{b_x}{\sqrt{b_x^2 + b_y^2}} \exp\left[-i \int R(b_x^2 + b_y^2)\mathrm{d}z\right]$$
$$E_y^{(\mathrm{o})} = \frac{b_y}{\sqrt{b_x^2 + b_y^2}} \exp\left[-i \int R(b_x^2 + b_y^2)\mathrm{d}z\right]$$
$$E_x^{(\mathrm{e})} = \frac{b_y}{\sqrt{b_x^2 + b_y^2}},$$
$$E_y^{(\mathrm{e})} = -\frac{b_x}{\sqrt{b_x^2 + b_y^2}},$$

Way to solve: change Ex, Ey to Eo, Ee.
Results show that the extraordinary mode remain unchanged.

$$\begin{cases} \dfrac{dE_x}{dz} + iRb_x(E_x b_x + E_y b_y) = 0 \\[2ex] \dfrac{dE_y}{dz} + iRb_y(E_x b_x + E_y b_y) = 0 \end{cases}$$

propagation equations



$$\begin{cases} E_x = E^{(o)} \cdot \dfrac{b_x}{\sqrt{b_x^2 + b_y^2}} + E^{(e)} \cdot \dfrac{b_y}{\sqrt{b_x^2 + b_y^2}} \\[2ex] E_y = E^{(o)} \cdot \dfrac{b_y}{\sqrt{b_x^2 + b_y^2}} - E^{(e)} \cdot \dfrac{b_x}{\sqrt{b_x^2 + b_y^2}} \end{cases}$$

divided into $E^{(o)}$ and $E^{(e)}$

$$\begin{cases} \dfrac{dE^{(o)}}{dz} \cdot \dfrac{b_x}{\sqrt{b_x^2+b_y^2}} + E^{(o)} \cdot \dfrac{d}{dz}\left(\dfrac{b_x}{\sqrt{b_x^2+b_y^2}}\right) + \dfrac{dE^{(e)}}{dz} \cdot \dfrac{b_y}{\sqrt{b_x^2+b_y^2}} + E^{(e)} \cdot \dfrac{d}{dz}\left(\dfrac{b_y}{\sqrt{b_x^2+b_y^2}}\right) + iRb_x \cdot E^{(o)} \cdot \sqrt{b_x^2+b_y^2} = 0 \\[2ex] \dfrac{dE^{(o)}}{dz} \cdot \dfrac{b_y}{\sqrt{b_x^2+b_y^2}} + E^{(o)} \cdot \dfrac{d}{dz}\left(\dfrac{b_y}{\sqrt{b_x^2+b_y^2}}\right) - \dfrac{dE^{(e)}}{dz} \cdot \dfrac{b_x}{\sqrt{b_x^2+b_y^2}} - E^{(e)} \cdot \dfrac{d}{dz}\left(\dfrac{b_x}{\sqrt{b_x^2+b_y^2}}\right) + iRb_y \cdot E^{(o)} \cdot \sqrt{b_x^2+b_y^2} = 0 \end{cases}$$

Geometrical Approx. $\Leftrightarrow$ Evironment ($\vec{B}_0 = B_0 \vec{b}$) slowly change

$$\Leftrightarrow \frac{d}{dz}\left(\frac{b_x}{\sqrt{b_x^2+b_y^2}}\right) = 0, \quad \frac{d}{dz}\left(\frac{b_y}{\sqrt{b_x^2+b_y^2}}\right) = 0$$

$$b_x \cdot \frac{dE^{(o)}}{dz} + b_y \cdot \frac{dE^{(e)}}{dz} + iRb_x \cdot \left(E^{(o)} \cdot b_x^2 + E^{(e)} \cdot b_x b_y + E^{(o)} \cdot b_y^2 - E^{(e)} \cdot b_x b_y\right) = 0$$

$$\Leftrightarrow \begin{cases} b_x \cdot \dfrac{dE^{(o)}}{dz} + b_y \cdot \dfrac{dE^{(e)}}{dz} + iRb_x \cdot (b_x^2 + b_y^2) \cdot E^{(o)} = 0 \\ b_y \cdot \dfrac{dE^{(o)}}{dz} - b_x \cdot \dfrac{dE^{(e)}}{dz} + iRb_y (b_x^2 + b_y^2) \cdot E^{(o)} = 0 \end{cases}$$

$$O \Rightarrow (b_x^2 + b_y^2) \frac{dE^{(o)}}{dz} + iR(b_x^2 + b_y^2)^2 E^{(o)} = 0 \Rightarrow E^{(o)} = C \cdot e^{-i\int R(b_x^2+b_y^2)dz}$$

$$E \Rightarrow \frac{dE^{(e)}}{dz} = 0 \Rightarrow E^{(e)} = C'$$

$E^{(o)}$ & $E^{(e)}$ are independent, no coupling

But when k nearly //B,

d(b_x)/dz is relatively violate:

$$\frac{|z-z_c|}{z_c} \ll 1$$

$$b_x = \theta(z - z_c)/z_c$$

$$\theta = \frac{\mathrm{d}b}{\mathrm{d}z/z_c}\Big|_{z=z_c}$$

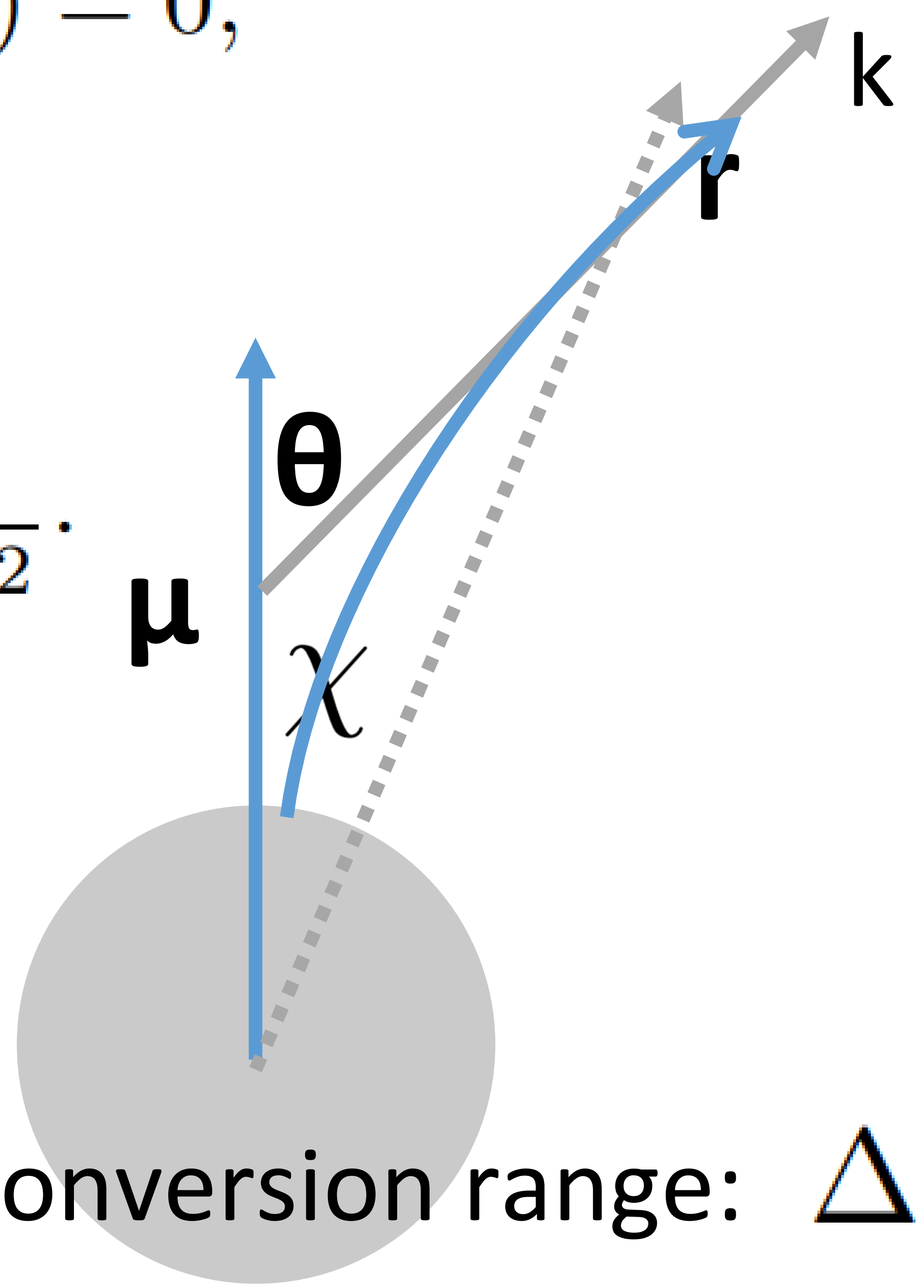
$$\frac{\mathrm{d}E_x}{\mathrm{d}z} + iRb_x(E_x b_x + E_y b_y) = 0,$$
$$\frac{\mathrm{d}E_y}{\mathrm{d}z} + iRb_y(E_x b_x + E_y b_y) = 0,$$

where

$$R \equiv \sum_\alpha \frac{\omega_{\mathrm{p}\alpha}^2}{2\omega c\gamma_\alpha^3(1-\beta_{0\alpha}b_z)^2}.$$

$$\frac{\mathrm{d}E_x}{\mathrm{d}u} + iu^2 E_x = -i\xi u E_y,$$
$$\frac{\mathrm{d}E_y}{\mathrm{d}u} + i\xi^2 E_y = -i\xi u E_x,$$

$$u \equiv (Rz_c\theta^2)^{1/3}(z - z_c)/z_c$$
$$\xi \equiv (Rz_c\theta^2)^{1/3} b_y/\theta$$



Conversion range: $\Delta u \sim 1$ i.e. $\Delta z/z_c \sim (Rz_c\theta^2)^{-1/3}$

Describing conversion:
(conversion degree)

$$\boldsymbol{E} = \alpha_1 \boldsymbol{E}^{(\mathrm{o})} + \alpha_2 \boldsymbol{E}^{(\mathrm{e})}$$

$$Q = \frac{|\boldsymbol{E} \cdot \boldsymbol{E}^{(\mathrm{e})*}|^2}{|\boldsymbol{E}|^2} = |\alpha_2|^2$$

$$\frac{\mathrm{d}E_x}{\mathrm{d}u} + iu^2 E_x = -i\xi u E_y,$$

$$\frac{\mathrm{d}E_y}{\mathrm{d}u} + i\xi^2 E_y = -i\xi u E_x,$$

$$u \equiv (R z_\mathrm{c} \theta^2)^{1/3} (z - z_\mathrm{c})/z_\mathrm{c}$$

$$\xi \equiv (R z_\mathrm{c} \theta^2)^{1/3} b_y/\theta$$

Easy case: $\xi \ll 1$

$$E_{x,y} = E_{0x,y} + \xi E_{1x,y} + \dots$$

Initial condition: pure O mode.

$$E_{0x} = C \exp(-iu^3/3), \; E_{1y} = -i\xi C \int_{-\infty}^{u} u \exp(-iu^3/3)$$

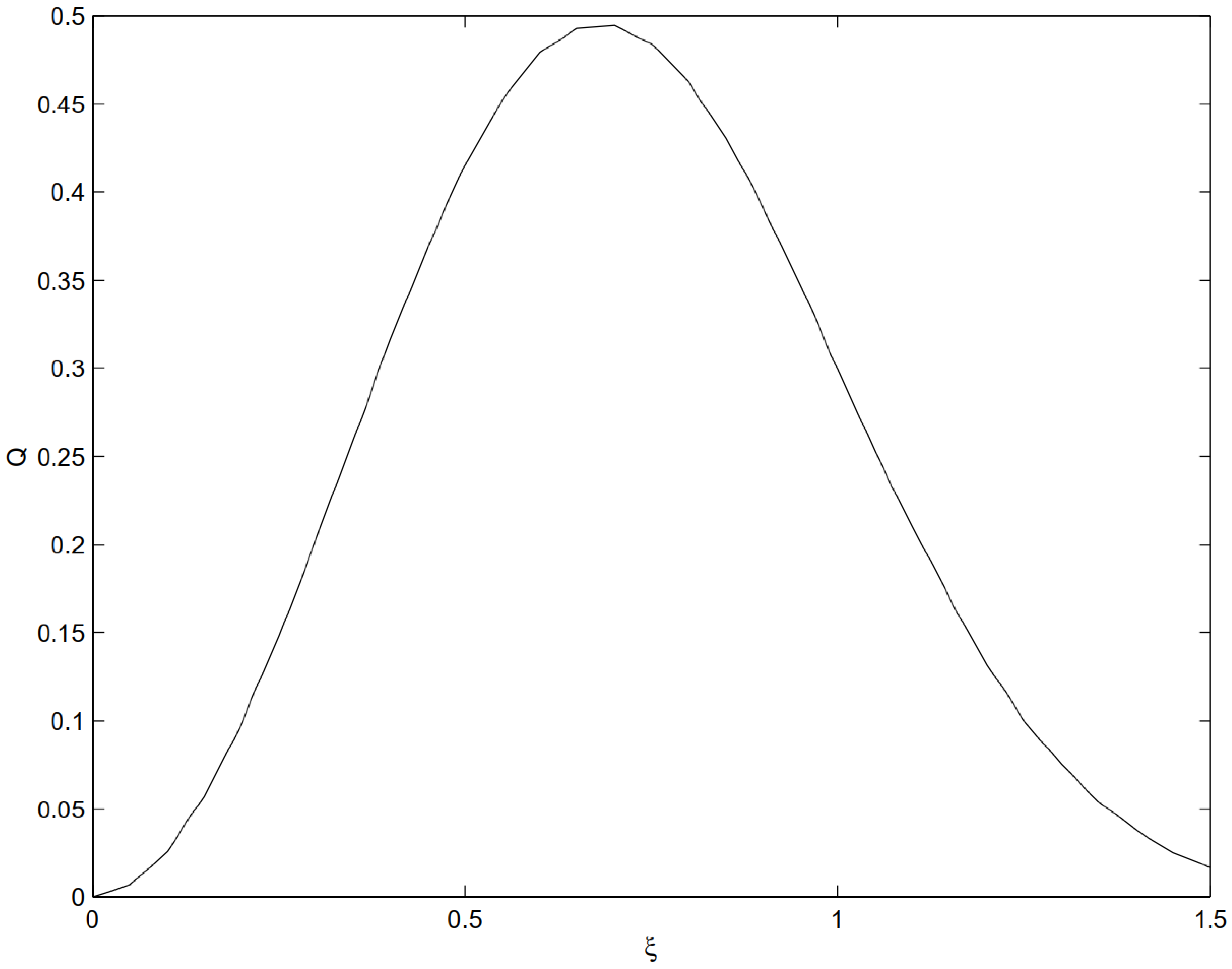
$$u \to \infty$$

$$E_x = C, \quad E_y = -3^{1/6}\Gamma(2/3) C\xi$$

$$Q = E_y^2 = 3^{1/3}\Gamma^2(2/3)\xi^2$$

as long as $\xi \ll 1$, Q increases with ξ

Numerical result: conversion degree can't exceed 0.5.



(3.2) In the limit $b_y \rightarrow 0$

B_0 is finite and $G \neq 0$

$$\frac{\mathrm{d}E_x}{\mathrm{d}z} + \frac{i\omega}{2c}[Ab_x(E_xb_x + E_yb_y) - BE_x + iGE_y] = 0,$$
$$\frac{\mathrm{d}E_y}{\mathrm{d}z} + \frac{i\omega}{2c}[Ab_y(E_xb_x + E_yb_y) - BE_y - iGE_x] = 0, \quad (12)$$

where

$$A \equiv \sum_\alpha \frac{\omega_{\mathrm{p}\alpha}^2}{\gamma_\alpha \omega'^2} \frac{\omega_{\mathrm{H}}^2}{\omega_{\mathrm{H}}^2 - \omega'^2};$$
$$B \equiv \sum_\alpha \frac{\omega_{\mathrm{p}\alpha}^2 \gamma_\alpha (1 - \beta_{0\alpha} b_z)^2}{\omega_{\mathrm{H}}^2 - \omega'^2};$$
$$G \equiv \sum_\alpha \frac{i(q_\alpha/e)(\omega_{\mathrm{H}}/\omega)\omega_{\mathrm{p}\alpha}^2(\beta_{0\alpha} - b_z)}{\omega_{\mathrm{H}}^2 - \omega'^2}.$$

$$\frac{\mathrm{d}a_x}{\mathrm{d}z} + iRb_x^2 a_x - Rga_y = 0,$$
$$\frac{\mathrm{d}a_y}{\mathrm{d}z} + Rga_x = 0,$$

$$E_{x,y} \equiv a_{x,y} \exp\left(i\frac{\omega}{2c}\int B\mathrm{d}z\right)$$
$$g \equiv G/A$$

Now R = A*ω / c

If adapting geometrical approximation:

$$\frac{\mathrm{d}a_x}{\mathrm{d}z} + iRb_x^2 a_x - Rga_y = 0,$$

$$\frac{\mathrm{d}a_y}{\mathrm{d}z} + Rga_x = 0,$$

$$E_{x,y} \equiv a_{x,y} \exp(i\tfrac{\omega}{2c} \textstyle\int B\mathrm{d}z)$$

$$g \equiv G/A$$

Now R = A*ω / c

$$\begin{aligned}
a_x^{(\mathrm{o})} &= \frac{i(b_x^2/2 + \sqrt{b_x^4/4 + g^2})}{\sqrt{g^2 + [b_x^2/2 + \sqrt{b_x^4/4 + g^2}]^2}} \\
&\quad \times \exp\left[-i \int R(b_x^2/2 + \sqrt{b_x^4/4 + g^2})\mathrm{d}z\right], \\
a_y^{(\mathrm{o})} &= \frac{g}{\sqrt{g^2 + [b_x^2/2 + \sqrt{b_x^4/4 + g^2}]^2}} \\
&\quad \times \exp\left[-i \int R(b_x^2/2 + \sqrt{b_x^4/4 + g^2})\mathrm{d}z\right], \\
a_x^{(\mathrm{e})} &= \frac{i(b_x^2/2 - \sqrt{b_x^4/4 + g^2})}{\sqrt{g^2 + [b_x^2/2 - \sqrt{b_x^4/4 + g^2}]^2}} \\
&\quad \times \exp\left[-i \int R(b_x^2/2 - \sqrt{b_x^4/4 + g^2})\mathrm{d}z\right], \\
a_y^{(\mathrm{e})} &= \frac{g}{\sqrt{g^2 + [b_x^2/2 - \sqrt{b_x^4/4 + g^2}]^2}} \\
&\quad \times \exp\left[-i \int R(b_x^2/2 - \sqrt{b_x^4/4 + g^2})\mathrm{d}z\right].
\end{aligned}$$

When quasi-longitudinal propagation: $\frac{|z-z_c|}{z_c} \ll 1$

$$\frac{da_x}{dz} + iRb_x^2 a_x - Rga_y = 0,$$
$$\frac{da_y}{dz} + Rga_x = 0,$$

$$\frac{da_x}{du} + iu^2 a_x = \eta a_y,$$
$$\frac{da_y}{du} = -\eta a_x,$$

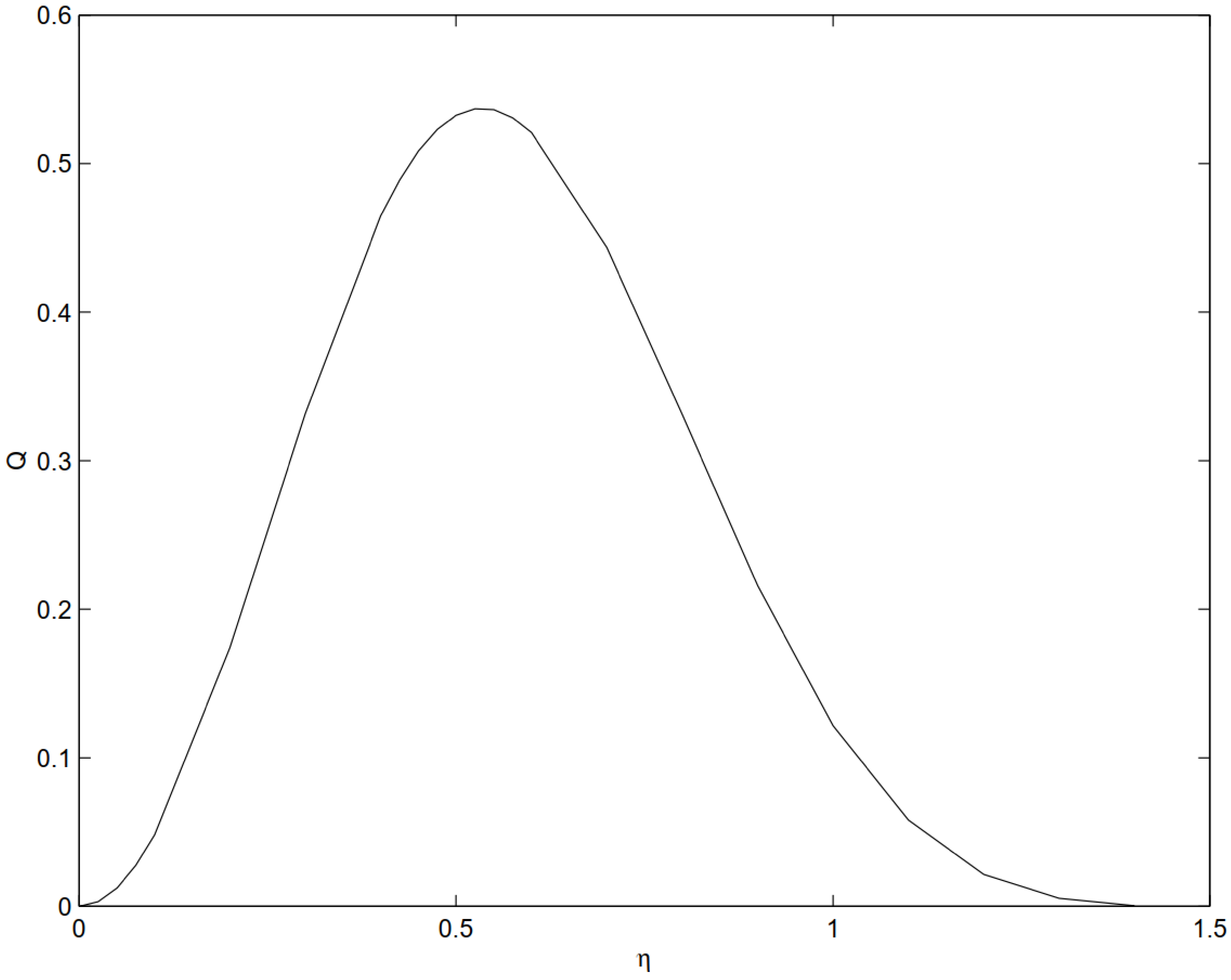
where $\eta \equiv (Rz_c/\theta)^{2/3} g.$

$$E_{x,y} \equiv a_{x,y} \exp(i\tfrac{\omega}{2c} \int B \mathrm{d}z)$$
$$g \equiv G/A$$

$$u \equiv (Rz_c\theta^2)^{1/3}(z - z_c)/z_c$$

Now R = A*ω / c

Numerical result: conversion degree can exceed 0.5.



(3.3) General Case, when quasi-longitudinal:

$$\frac{\mathrm{d}E_x}{\mathrm{d}z} + \frac{i\omega}{2c}[Ab_x(E_x b_x + E_y b_y) - BE_x + iGE_y] = 0,$$

$$\frac{\mathrm{d}E_y}{\mathrm{d}z} + \frac{i\omega}{2c}[Ab_y(E_x b_x + E_y b_y) - BE_y - iGE_x] = 0, \quad (12)$$

where

$$A \equiv \sum_\alpha \frac{\omega_{\mathrm{p}\alpha}^2}{\gamma_\alpha \omega'^2} \frac{\omega_{\mathrm{H}}^2}{\omega_{\mathrm{H}}^2 - \omega'^2};$$

$$B \equiv \sum_\alpha \frac{\omega_{\mathrm{p}\alpha}^2 \gamma_\alpha (1 - \beta_{0\alpha} b_z)^2}{\omega_{\mathrm{H}}^2 - \omega'^2};$$

$$G \equiv \sum_\alpha \frac{i(q_\alpha/e)(\omega_{\mathrm{H}}/\omega)\omega_{\mathrm{p}\alpha}^2(\beta_{0\alpha} - b_z)}{\omega_{\mathrm{H}}^2 - \omega'^2}.$$

$$\frac{\mathrm{d}a_x}{\mathrm{d}u} + iu^2 a_x + i\xi u a_y - \eta a_y = 0,$$

$$\frac{\mathrm{d}a_y}{\mathrm{d}u} + i\xi u a_x + i\xi^2 a_y + \eta a_x = 0.$$

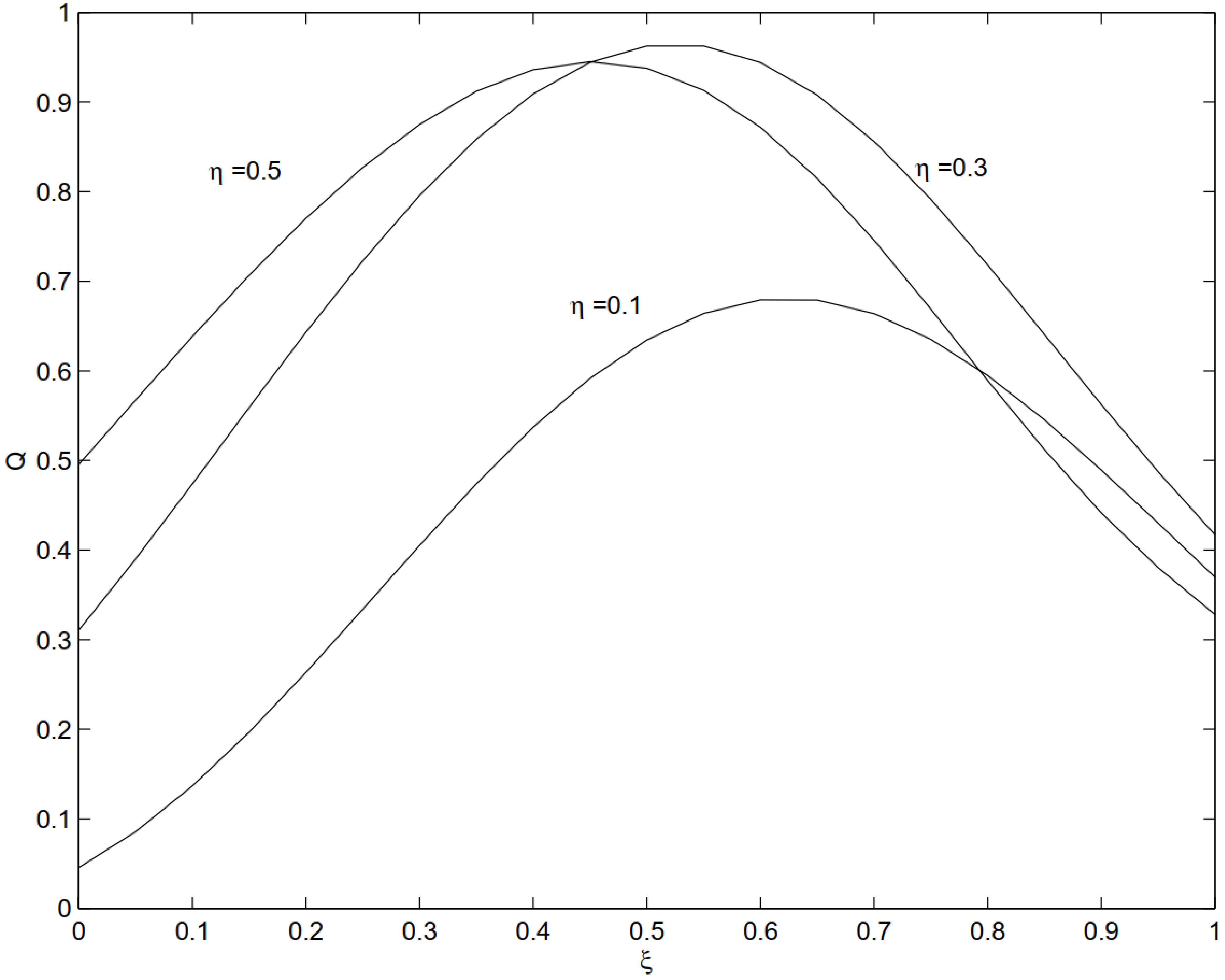
where $\eta \equiv (Rz_{\mathrm{c}}/\theta)^{2/3} g.$ $\quad g \equiv G/A$

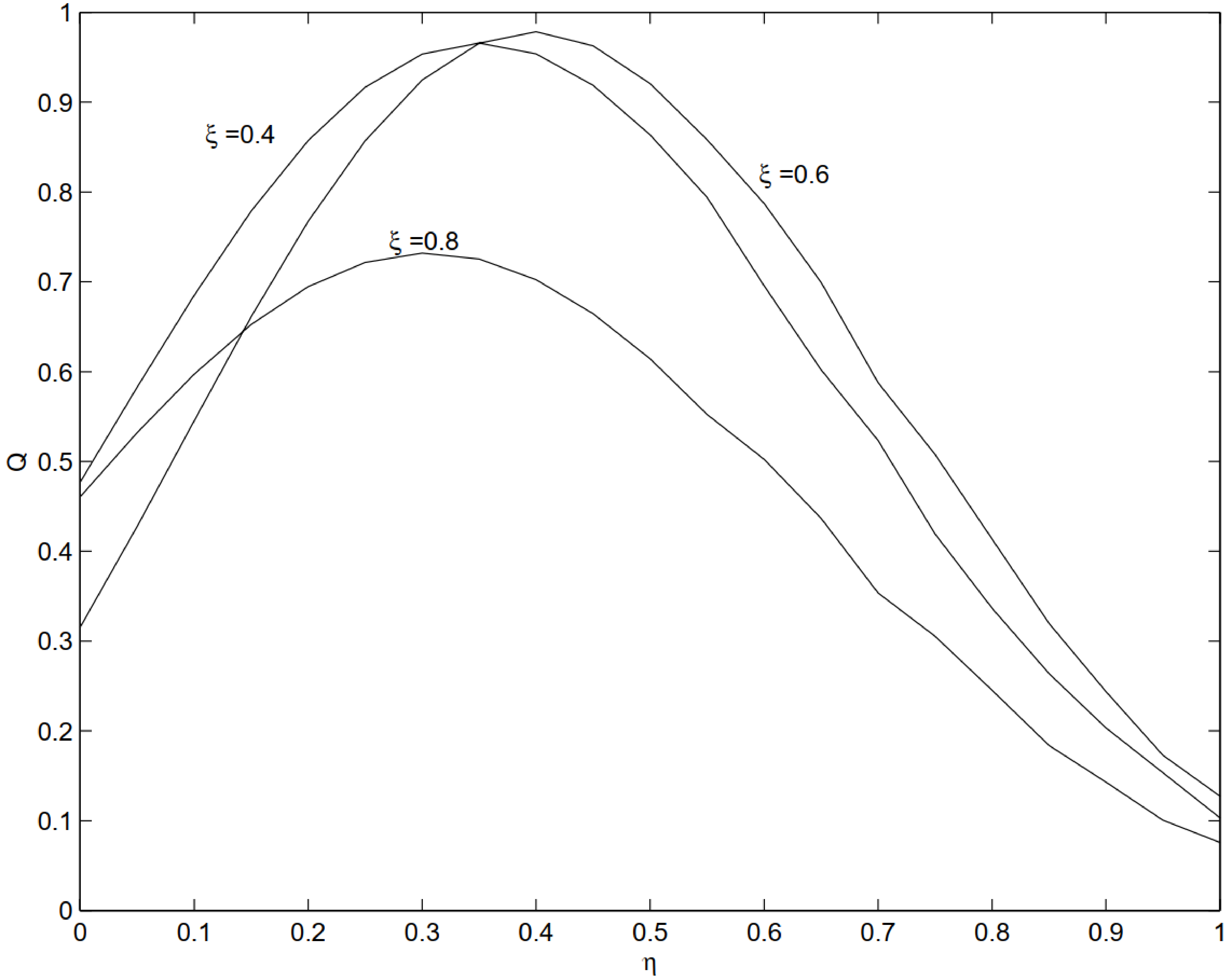
$$E_{x,y} \equiv a_{x,y} \exp(i\tfrac{\omega}{2c} \textstyle\int B \mathrm{d}z)$$

$$u \equiv (Rz_{\mathrm{c}}\theta^2)^{1/3}(z - z_{\mathrm{c}})/z_{\mathrm{c}}$$

$$\xi \equiv (Rz_{\mathrm{c}}\theta^2)^{1/3} b_y/\theta$$

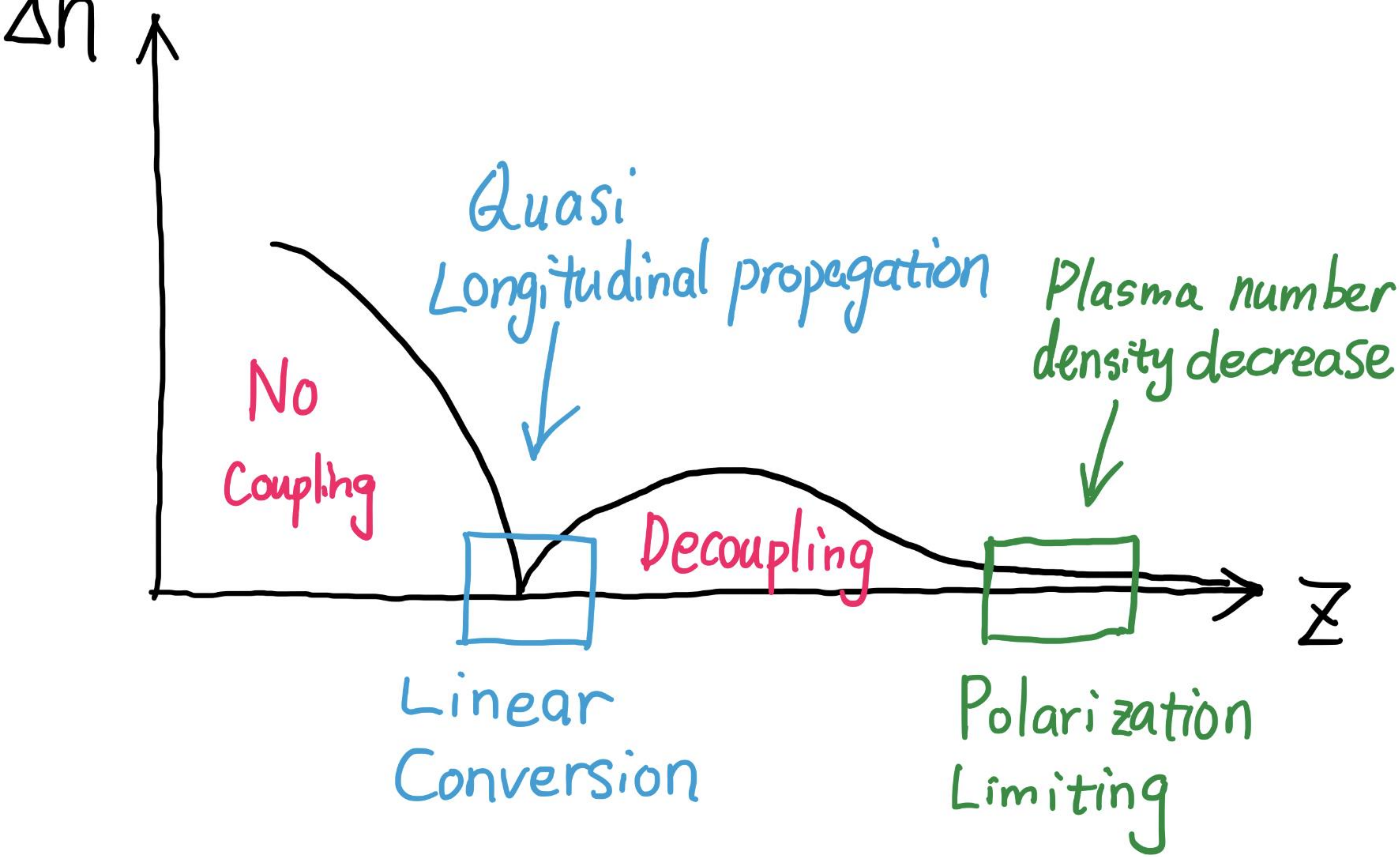
Numerical result: conversion degree can approach 1.





$0.1 \lesssim \xi \lesssim 1 \quad 0.1 \lesssim \eta \lesssim 1$

(4) Polarization-limiting effect



Easy case: $b_x = \text{const.},\ b_y \propto z$ plasma number density decreases as z^{-3}

$$\frac{\mathrm{d}E_x}{\mathrm{d}z} + \frac{i\omega}{2c}[Ab_x(E_x b_x + E_y b_y) - BE_x + iGE_y] = 0,$$
$$\frac{\mathrm{d}E_y}{\mathrm{d}z} + \frac{i\omega}{2c}[Ab_y(E_x b_x + E_y b_y) - BE_y - iGE_x] = 0, \quad (12)$$

where

$$A \equiv \sum_\alpha \frac{\omega_{\mathrm{p}\alpha}^2}{\gamma_\alpha \omega'^2} \frac{\omega_{\mathrm{H}}^2}{\omega_{\mathrm{H}}^2 - \omega'^2};$$
$$B \equiv \sum_\alpha \frac{\omega_{\mathrm{p}\alpha}^2 \gamma_\alpha (1 - \beta_{0\alpha} b_z)^2}{\omega_{\mathrm{H}}^2 - \omega'^2};$$
$$G \equiv \sum_\alpha \frac{i(q_\alpha/e)(\omega_{\mathrm{H}}/\omega)\omega_{\mathrm{p}\alpha}^2(\beta_{0\alpha} - b_z)}{\omega_{\mathrm{H}}^2 - \omega'^2}.$$

$$\frac{\mathrm{d}E_x}{\mathrm{d}w} - is(w)wE_x - is(w)\mu E_y = 0,$$
$$\frac{\mathrm{d}E_y}{\mathrm{d}w} - is(w)\mu E_x - is(w)\mu^2/wE_y = 0. \quad (28)$$

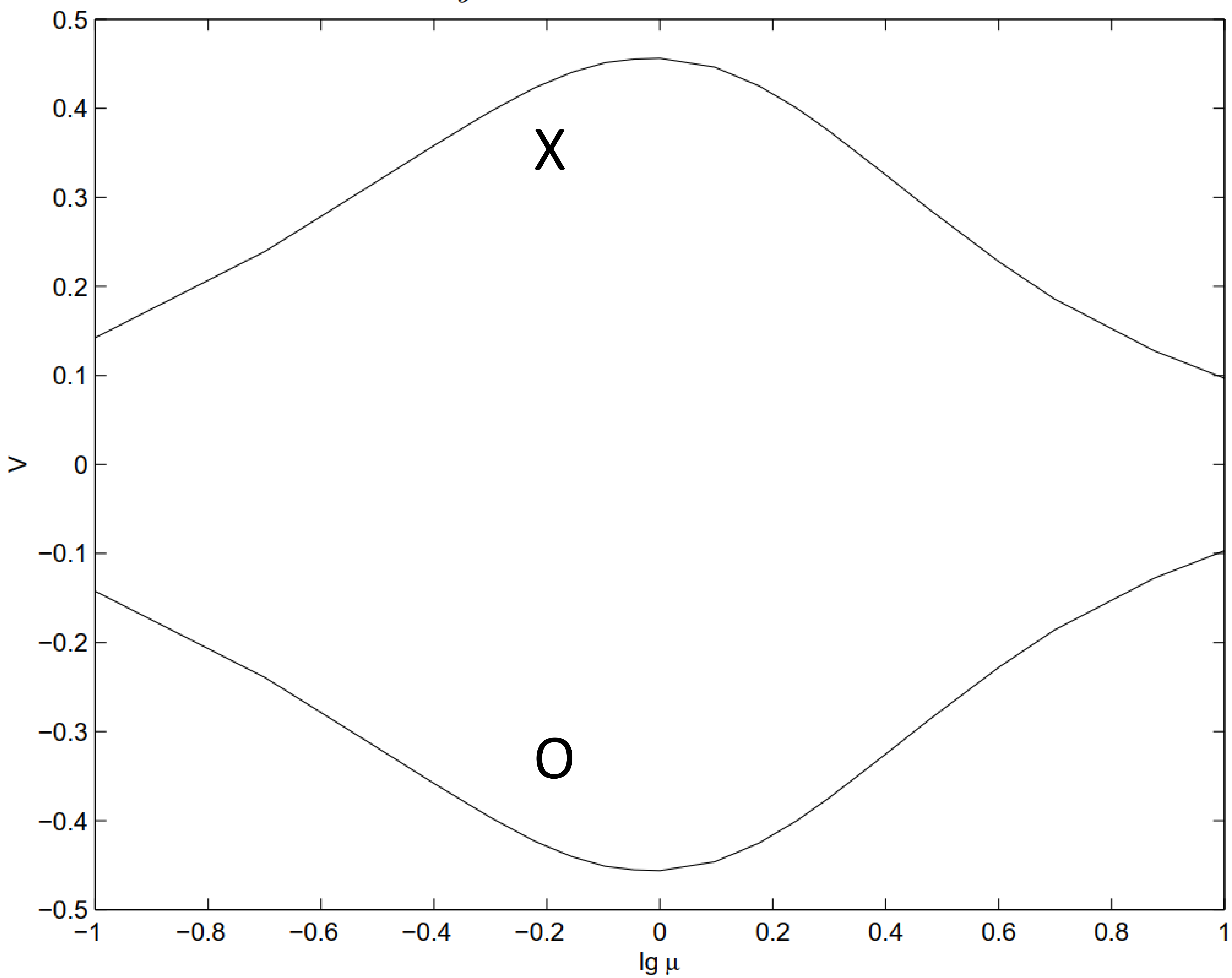
Here $w \equiv z_{\mathrm{p}}/z$, z_{p} is the polarization-limiting radius determined by the following relation:

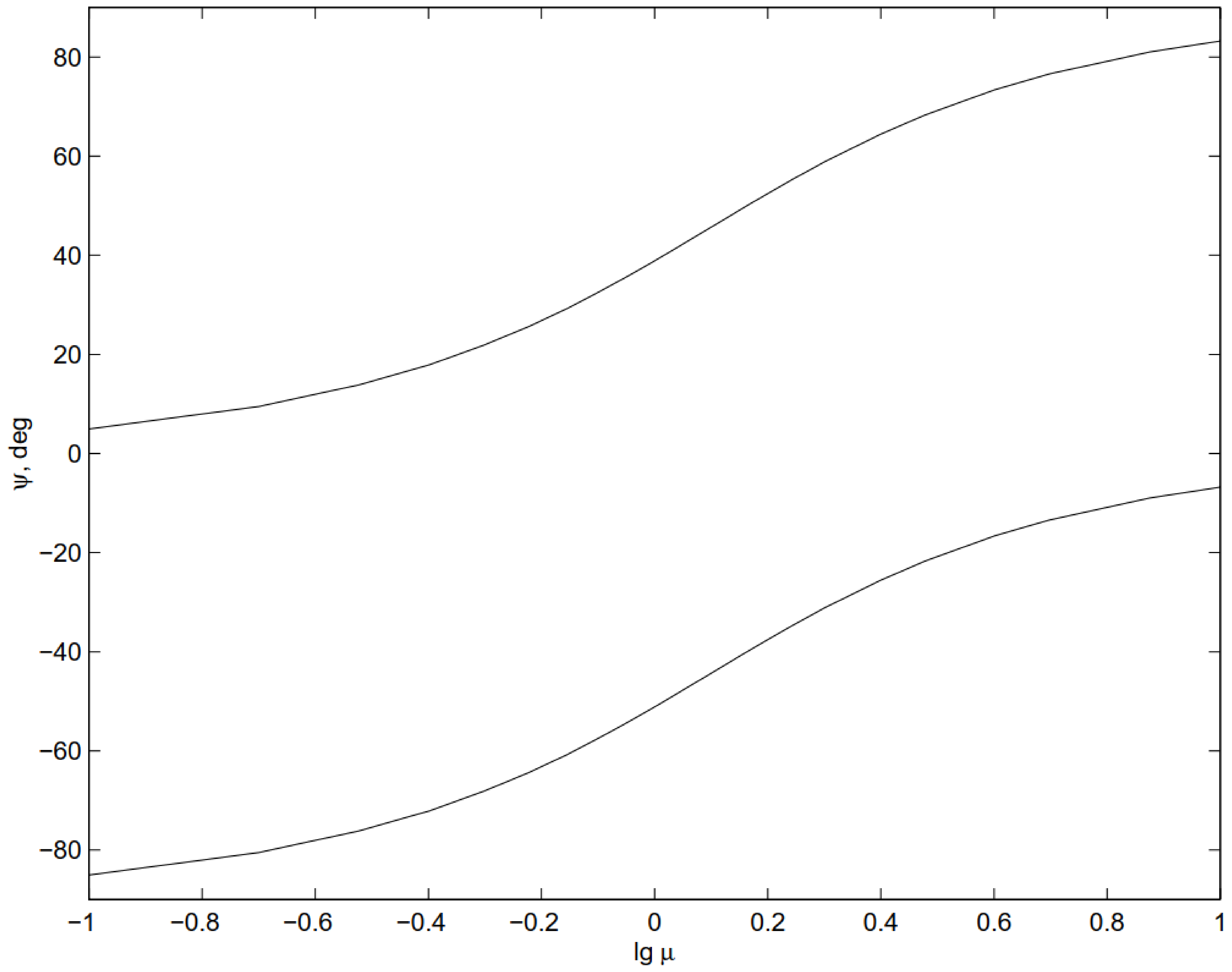
$$R(z_{\mathrm{p}})[b_x^2 + b_y^2(z_{\mathrm{p}})]z_{\mathrm{p}} = 1,$$

$$\mu \equiv (b_y/b_x)_{z=z_{\mathrm{p}}},\ s(w) \equiv \frac{1+\mu^2}{(1+\mu^2/w^2)^2}.$$

Numerical result: O and X mode get circular polarization **(of different signs).**

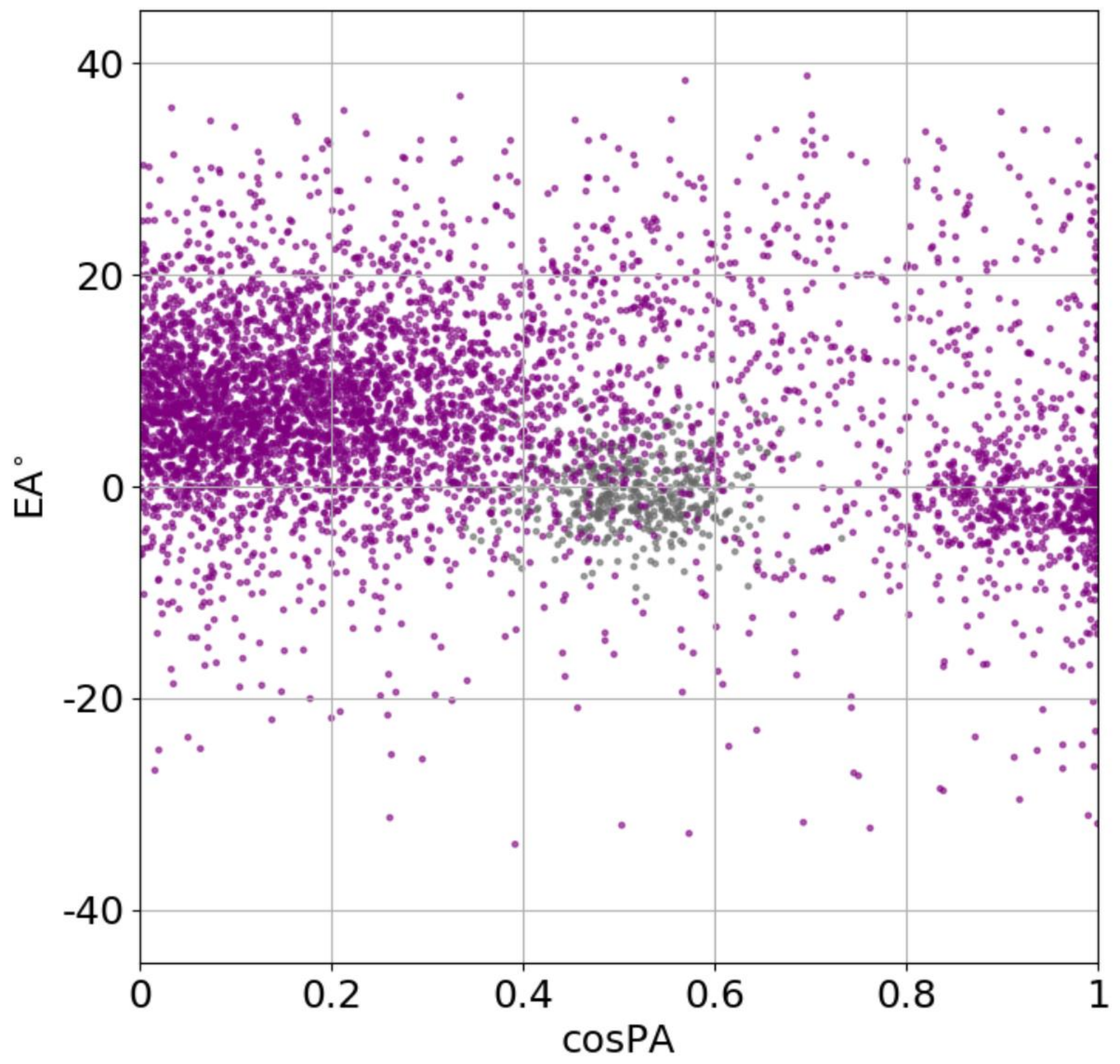
$$V \equiv \frac{i(E_x^* E_y - E_x E_y^*)}{E_x E_x^* + E_y E_y^*}$$

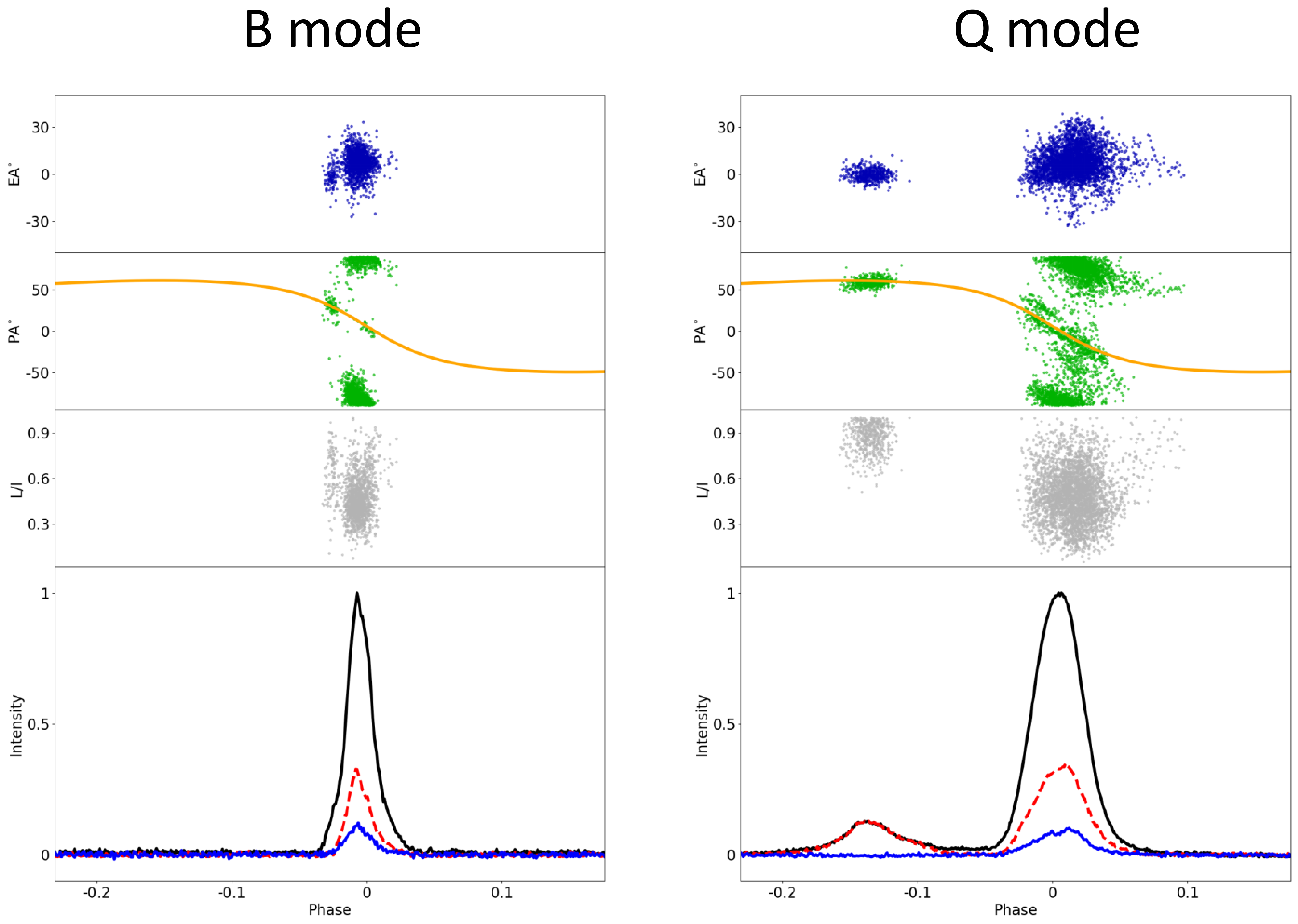




$$\tan 2\psi \equiv \frac{E_y E_x^* + E_x E_y^*}{E_x E_x^* - E_y E_y^*}.$$

私货环节：FAST B0943+10 results





If OPMs arise from propagation, and longer propagation leads to more X mode, then during mode switch, maybe plasma number density is modified, thus OPMs and profiles all change.

Refraction ➔ Quasi-longitudinal propagation ➔ Modes linear coupling ➔ O/X modes conversion

Thank you for your attention ☺