A model for nulling & mode changing in pulsars.

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Phenomena { mode changing : 不同时段, integrated prof 变
nulling : 有的时段 pulse 看不到

Same ? ← single class of reasons

时标: a few pulse periods

properties → hours/days...

nulling { 轻微
严重

reasons for nulling { "microphysics" in radiation mechanism
↕ or? both?
whole magnetosphere's change

PSR B1931+24 & J1832+0029
(Kramer et al. 2006, Lyne 2009)
→ $\dot{P}$ 不同, 在 nulling & 非 nulling 态

至少有一部分 nulling 伴随着磁层整体变化。

This paper: qualitative (semi-quantitative) model to explain nulling & mode change, and the $\dot{P}$ change.

⇒ $\dot{\Omega}$ 的变化不能仅因为有无 radio emission 导致

~变化 50% B1931+24

less when radio invisible

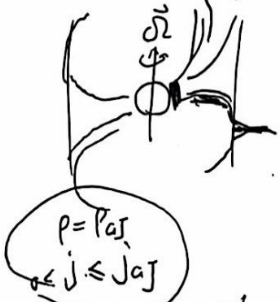
days of nulling

many other psrs: ~hours or less

$\dot{P}$ 变化不易测 (timing)

Globular changes: in magnetosphere structure

• Recap: psr magnetosphere




accelerating E only in geometrically very small regions

most magnetosphere: force-free

如何影响 $\dot{P}$: 改变 $\dot{E}$ { 改变 j & B

改磁层位形 or 改 current 分布

modeswitch & nulling:
switches between some quasi-stable states. { ① 不同的 closed field line zone
② 不同的 open field line 区 current distribution
③ both

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• Recap: spin-down 能量分配

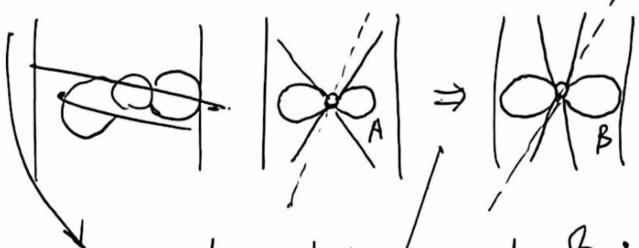
$E_{spindown} = \frac{1}{2} I \Omega^2$ , $\dot{E} = I\Omega\dot{\Omega}$ { $<10^{-3}$ ⇒ radio emission
$\sim 10^{-1}$ ⇒ HE emission/cold particle wind (non-radiative)

When will nullings happen?

① coherent condition broken

② coherent emission generated elsewhere.

② 的一种情形: radio emission zone shrink




correlation between width & $\dot{P}$ in mode-switching pulsars?

具体的model: simplified
- aligned rotator
- dipole $\vec{B}$

aim: to prove modest change in magneto $\Longrightarrow$ large variations in $\dot{P}$

②.1 Shrinking-expanding corotating zone

A: larger opening angle.


$\alpha \sim \frac{3}{2}\theta \propto \theta_{pc}$

$\varpi$: $\rho_i$ 的一种写法

formulae:

① a. $B$ 表示法

cylindrical: $(\varpi, \phi, Z)$

$$\vec{B} = \frac{\nabla\Psi \times \vec{e}_\phi}{\varpi} + \frac{4\pi}{c}\cdot\frac{I}{\varpi}\vec{e}_\phi$$

极向场　　环向场


$$(B_\varpi, B_\phi, B_z) = \left(-\frac{1}{\varpi}\partial_z\Psi,\ \frac{4\pi}{c}\frac{I}{\varpi},\ \frac{1}{\varpi}\partial_\varpi\Psi\right)$$

$$\nabla\times\vec{B} = \begin{vmatrix} \frac{1}{\varpi}\vec{e}_\varpi & \vec{e}_\phi & \frac{1}{\varpi}\vec{e}_z \\ \frac{\partial}{\partial\varpi} & \frac{\partial}{\partial\phi} & \frac{\partial}{\partial z} \\ -\frac{1}{\varpi}\partial_z\Psi & \frac{4\pi}{c}\frac{I}{\varpi} & \frac{1}{\varpi}\partial_\varpi\Psi \end{vmatrix}$$

$$= \frac{1}{\varpi}\vec{e}_\varpi\left(\frac{1}{\varpi}\partial_\phi\partial_\varpi\Psi - \frac{4\pi}{c}\frac{1}{\varpi}\partial_z I\right) - \vec{e}_\phi\left(-\frac{1}{\varpi^2}\partial_\varpi\Psi + \frac{1}{\varpi}\partial_\varpi^2\Psi + \frac{1}{\varpi}\partial_z^2\Psi\right) + \frac{1}{\varpi}\vec{e}_z\left(\frac{4\pi}{c}\frac{1}{\varpi}I + \frac{4\pi}{c}\frac{1}{\varpi}\partial_\varpi I + \frac{1}{\varpi}\partial_z\partial_\phi\Psi\right)$$

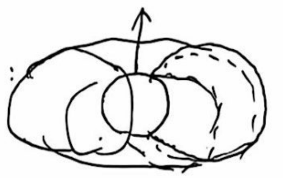
cylindrical $(\rho, \phi, z)$　　$\frac{S}{\rho}\vec{e}_\phi$

$$\vec{B} = \frac{\nabla\Psi\times\vec{e}_\phi}{\rho} + \frac{4\pi}{c}\cdot\frac{I}{\rho}\cdot\vec{e}_\phi$$

poloidal　　toroidal

$$(B_\rho, B_\phi, B_z) = \left(-\frac{1}{\rho}\partial_z\Psi,\ \frac{4\pi}{c}\frac{I}{\rho},\ \frac{1}{\rho}\partial_\rho\Psi\right)$$

磁面:



magnetic surface.

空间中的位形: 用一个函数$\Psi$表示

$\Psi(\rho, z) = \text{Const}$ 表示一个磁面/磁力线 (轴对称)

flux $\Phi_{mag} = 2\pi\Psi$

参考 BGI书

依定义, 磁面的法向 ⊥ 磁场

$\Rightarrow \vec{B}\cdot\nabla\Psi = 0$

同时, $\Psi$满足, $\forall\,\Phi_0 = \Phi_0(\phi)$, 有

$\nabla\Psi\cdot\nabla\Phi_0 = 0$

$$\mathbb{B} = \underbrace{\nabla\Psi \times \nabla\Phi_0}_{\text{Euler potential}} + g\cdot\nabla\Phi_0 \quad \text{general form}$$

$g\cdot\nabla\Phi_0 \rightarrow$ 可以有环向分量

$g = g(\rho, z)$.

$$\frac{c}{4\pi}\left[\vec{E}\times\mathbb{B}\right]_{pol} = -\frac{\Omega}{c} I \vec{B}_{pol}$$

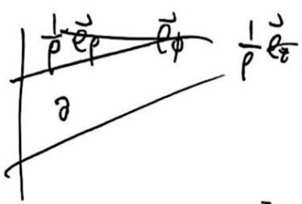


$$\nabla\cdot\mathbb{B} = \nabla\cdot(\nabla\Psi\times\nabla\Phi_0) + \cancel{\nabla g\cdot\nabla\Phi_0}\ \nabla\cdot(g\cdot\nabla\Phi_0)$$

$$\nabla\Psi = \frac{\partial\Psi}{\partial\rho}\vec{e}_\rho + \frac{1}{\rho}\frac{\partial\Psi}{\partial\phi}\vec{e}_\phi + \frac{\partial\Psi}{\partial z}\vec{e}_z$$

$$= \frac{\partial\Psi}{\partial\rho}\vec{e}_\rho + 0 + \frac{\partial\Psi}{\partial z}\vec{e}_z$$

$$\nabla\Phi_0 = \frac{1}{\rho}\frac{\partial\Phi}{\partial\phi}\vec{e}_\phi$$





$$\nabla\Psi\times\nabla\Phi_0 = \frac{1}{\rho}\frac{\partial\Psi}{\partial\rho}\cdot\frac{\partial\Phi}{\partial\phi}\cdot\vec{e}_z - \frac{1}{\rho}\frac{\partial\Psi}{\partial z}\cdot\frac{\partial\Phi}{\partial\phi}\vec{e}_\rho$$

$$\nabla\cdot(\cdots) = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\cancel{\rho}\cancel{\frac{1}{\rho}}\frac{\partial\Psi}{\partial z}\cdot\frac{\partial\Phi}{\partial\phi}\right) + \frac{\partial}{\partial z}\left(\frac{1}{\rho}\frac{\partial\Psi}{\partial\rho}\cdot\frac{\partial\Phi}{\partial\phi}\right)$$

$$= -\frac{1}{\rho}\frac{\partial\Phi}{\partial\phi}\cdot\frac{\partial^2\Psi}{\partial\rho\partial z} + \frac{1}{\rho}\frac{\partial^2\Psi}{\partial\rho\partial z}\cdot\frac{\partial\Phi}{\partial\phi} = 0$$

$$\nabla\cdot(g\cdot\nabla\Phi_0) = \cancel{\nabla g\cdot\nabla\Phi_0}\ \nabla\cdot\left(\frac{g}{\rho}\cdot\frac{\partial\Phi}{\partial\phi}\vec{e}_\phi\right)$$

$$= \frac{1}{\rho}\frac{\partial}{\partial\varphi}\left(\frac{g}{\rho}\cdot\frac{\partial\Phi}{\partial\phi}\right) = \frac{g}{\rho^2}\cdot\frac{\partial^2\Phi}{\partial\phi^2} = 0$$

取 $\Phi_0 = \phi$ , $\nabla\Phi_0 = \frac{1}{\rho}\vec{e}_\phi$

$$\Rightarrow \mathbb{B} = \frac{1}{\rho}\nabla\Psi\times\vec{e}_\phi + \frac{1}{\rho}\cdot g\cdot\vec{e}_\phi$$

$$\Downarrow$$

$$\mathbb{B} = \frac{\nabla\Psi\times\vec{e}_\phi}{\rho} + \frac{4\pi}{c}\cdot\frac{I}{\rho}\cdot\vec{e}_\phi$$

dipole: $\Psi \equiv \mu\cdot\frac{\rho^2}{(\rho^2+z^2)^{3/2}}$

$$\vec{E} = -\frac{(\vec{\Omega}\times\vec{r})}{c}\times\mathbb{B} = -\frac{1}{c}\left(\Omega\vec{e}_z\times(\rho\vec{e}_\rho + z\hat{e}_z)\right)\times\mathbb{B}$$

$$= -\frac{1}{c}\Omega\rho\cdot\vec{e}_\phi\times\left(\frac{1}{\rho}\nabla\Psi\times\vec{e}_\phi + \frac{1}{\rho}g\vec{e}_\phi\right)$$

$$= -\frac{1}{c}\Omega\nabla\Psi$$

$$\Psi_{last} = \Psi|_{z=0} = \mu\cdot\frac{1}{\rho}$$

$$\cancel{\Rightarrow P_r = -\frac{I\Omega}{c}\frac{1}{\rho r}(}$$

$$B_{pol} = -\frac{1}{\rho}\partial_z\Psi\cdot\vec{e}_\rho + \frac{1}{\rho}\partial_\rho\Psi\cdot\vec{e}_z$$

$$\vec{P}_{pol} = \frac{\Omega}{c}I\cdot\frac{1}{\rho}\left(\partial_z\Psi\vec{e}_\rho - \partial_\rho\Psi\cdot\vec{e}_z\right)$$


$$P_r = -\frac{I\Omega}{c}\frac{1}{\rho\cdot r}\cdot\left(z\cdot\partial_\rho\Psi - \rho\cdot\partial_z\Psi\right)$$

Energy loss: $dW = -r^2 P_r\cdot d\omega \rightarrow$ solid angle

$$\Rightarrow \frac{dW}{d\omega} = \frac{I\Omega}{c}\cdot\frac{r}{\rho}\cdot\left(z\cdot\partial_\rho\Psi - \rho\partial_z\Psi\right)$$

$$W = \int_{4\pi}\frac{dW}{d\omega}\cdot d\omega$$

$$= \int_{4\pi}\frac{dW}{d\omega}\cdot\sin\theta\, d\theta\, d\phi$$

$$\frac{\rho}{\sqrt{z^2+\rho^2}}$$


$\rho = r\sin\theta$ , $r = \sqrt{\rho^2+z^2}$

$\phi = \phi$ , $\theta = \arctan\frac{\rho}{z}$

$z = r\cos\theta$ , $\phi = \phi$

$$d\theta = \frac{1}{1+\left(\frac{\rho}{z}\right)^2}\left(\frac{d\rho}{z} - \frac{\rho dz}{z^2}\right)$$

$$= \frac{1}{z^2+\rho^2}(z d\rho - \rho dz)$$

$$W=\int_{4\pi}\frac{I\Omega}{c}\cdot\frac{r}{\rho}\left(z\partial_\rho\Psi-\rho\partial_z\Psi\right)\cdot\frac{\rho}{\sqrt{z^2+\rho^2}}\cdot\frac{1}{z^2+\rho^2}\cancel{(z\,d\rho-\rho\,dz)}$$

$\sin\theta\,d\theta\,d\phi$

$d\rho=r\cos\theta\cdot d\theta$

$z=\cancel{\rho\cos\theta}\ r\cos\theta$

$\rho=r\sin\theta$

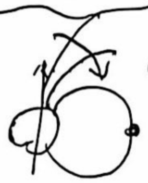
$$\partial_\rho\Psi=(\partial_r\Psi)\frac{\partial r}{\partial\rho}+(\partial_\theta\Psi)\frac{\partial\theta}{\partial\rho}=(\partial_r\Psi)\frac{1}{\sin\theta}+(\partial_\theta\Psi)\cdot\frac{1}{r\cos\theta}$$

$$\partial_z\Psi=(\partial_r\Psi)\frac{\partial r}{\partial z}+(\partial_\theta\Psi)\cdot\frac{\partial\theta}{\partial z}=(\partial_r\Psi)\frac{1}{\cos\theta}-(\partial_\theta\Psi)\cdot\frac{1}{r\sin\theta}$$

$$W=\int_{4\pi}\frac{I\Omega}{c}\cdot\cancel{\frac{1}{\sin\theta}}\cdot\left(r\cos\theta\cdot\partial_\rho\Psi-r\sin\theta\,\partial_z\Psi\right)\cancel{\sin\theta}\,d\theta\,d\phi$$

$$=\int_{4\pi}\frac{I\Omega}{c}\cdot\left(\underline{r\cdot\frac{\cos\theta}{\sin\theta}\cdot\partial_r\Psi+\partial_\theta\Psi}-\underline{r\cdot\frac{\sin\theta}{\cos\theta}\cdot\partial_r\Psi}+\partial_\theta\Psi\right)d\theta\cdot d\phi$$

$$=\boxed{2\int_{\cancel{\Psi_{\theta=0}}\,0}^{\Psi}\frac{2\pi}{c}I\Omega_F\,d\Psi \qquad I=I(\Psi)}$$



open 区全积了

$$W=|W_{md}|\int_0^{\Psi_{last}}S\,d\Psi$$

$S=-\cancel{\Psi(2}-\Psi(2-\frac{\Psi}{\Psi_{last}})$

$-\int_0^{\Psi_{last}}\Psi(2-\frac{\Psi}{\Psi_{last}})\,d\Psi$ ?

$-\cancel{2}\Psi_{last}^2+\frac{1}{3}\cdot\Psi_{last}^2$

$$S=-\left(\Psi_{last}-\frac{\Psi^2}{\Psi_{last}}\right)$$

$\Psi_{last}^2-\frac{1}{3}$

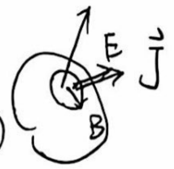
$$I\propto-\Psi\left(2-\frac{\Psi}{\Psi_{last}}\right)$$ Michel 1973b

$J=2\pi I$

$\Rightarrow \cancel{W\propto}$

$W\propto\Psi_{last}^2\propto\cancel{r_0^{-2}}\ \rho_{last}^{-2}$

$\theta_{pc}=\sqrt{\frac{R_{NS}}{\rho_{last}}}$ $W\propto\theta_{pc}^4\propto\alpha^4$




$P=I\cdot U$

2.2 $\rho_{last}$ 不变, $\vec{J}$ 变 $\Rightarrow W\sim\int\vec{J}\cdot\vec{E}\,\cancel{ds}\,dl$

$\sim\int J\cdot\Omega B\cdot r\,d\cancel{l}/c$

$\sim\cancel{J\cdot\Omega B\,\pi r_p^2}$

$\propto J\cdot\Omega$

$$S=\frac{4\pi}{c}\cdot\frac{R_{LC}}{\Psi_0}\cdot I=S(\Psi)$$

Pulsar Equation Michel 1973a,b

$$\Psi_{zz}+\Psi_{\rho\rho}-\frac{1}{\rho}\left(\frac{R_{LC}^2+\rho^2}{R_{LC}^2-\rho^2}\right)\cdot\Psi_\rho+\frac{\cancel{\Psi}\,S'\cdot S}{R_{LC}^2-\rho^2}=0$$

~~有~~ $J \propto \Omega$ (Blandford & Znajek (1977)

$\Rightarrow W \propto J^2 \xrightarrow{?} \propto \alpha^{4.5}$

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- . .

△ Discussion:

① If nulling/mode change pure geometrical?

→ Not really...


For this kind of psr, change of magnetospheric paras may cease radio emission...

② mode switching pulsars' $\dot{P}$ change?

↓ measurements... with high timing noise.

③ Why states are quasi-stable? ($\gg P_1$)

total energy $E = E(R_Y, J, \ldots)$


local minima?

↓

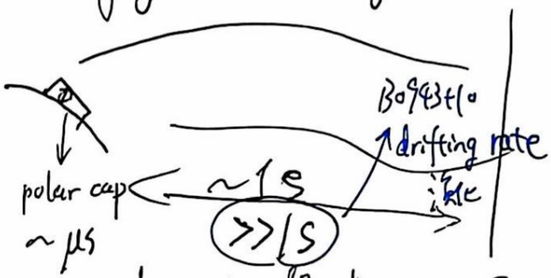
different states.

J distribution change $\Leftarrow V_{polarcap}$ change

With psr aging, $V_{polarcap}$ ↓ (vacuum)

small potential variation leads to larger changes

④ psr magnetosphere: highly non-linear system.




dynamics & adjustment?

↳ strange attractor (B0943?)

↳ quasi-periodically

⑤ RRAT: pure geometrical?

→ 统计/测一下RRAT的几何?

https://rratalog.github.io/rratalog

~~好像~~ 没人干过...

~~⑥~~

$$\boxed{\Psi = \Psi_0 \cdot \frac{z}{r} = \Psi_0 \cdot \frac{z}{\sqrt{\rho^2+z^2}}}$$

$$\Psi_z = \Psi_0 \cdot \frac{\sqrt{\rho^2+z^2} - z\cdot\frac{z}{\sqrt{\rho^2+z^2}}}{\rho^2+z^2} = \Psi_0 \frac{\rho^2}{(\rho^2+z^2)^{\frac{3}{2}}}$$

$$\Psi_{zz} = \Psi_0 \cdot \rho^2 \cdot \frac{\left(-\frac{3}{2}\right)}{(\rho^2+z^2)^{\frac{5}{2}}} \cdot 2z$$

$$= -\Psi_0 \cdot \rho^2 \cdot \frac{3z}{(\rho^2+z^2)^{\frac{5}{2}}}$$

$$\Psi_\rho = \Psi_0 \cdot z \cdot \frac{-\frac{1}{2}}{(\rho^2+z^2)^{3/2}} \cdot 2\rho$$

$$= -\Psi_0 \cdot \frac{\rho z}{(\rho^2+z^2)^{3/2}}$$

$$\Psi_{\rho\rho} = -\Psi_0 \cdot \frac{z\cdot(\rho^2+z^2)^{3/2} - \rho z\cdot\frac{3}{2}\cdot(\rho^2+z^2)^{\frac{1}{2}}\cdot 2\rho}{(\rho^2+z^2)^3}$$

$$= -\Psi_0 \frac{z\cdot(\rho^2+z^2) - 3\rho^2 z}{(\rho^2+z^2)^{5/2}}$$

$$= -\Psi_0 \frac{z^3 - 2\rho^2 z}{(\rho^2+z^2)^{5/2}}$$

$$\Psi_{zz} + \Psi_{\rho\rho} = -\Psi_0 \cdot \frac{z^3 + \rho^2 z}{(\rho^2+z^2)^{5/2}}$$

$$= -\Psi_0 \cdot \frac{z}{(\rho^2+z^2)^{3/2}}$$

$$-\Psi_0 \cdot \frac{z}{(\rho^2+z^2)^{3/2}} + \Psi_0 \cdot \left(\frac{R_{LC}^2+\rho^2}{R_{LC}^2-\rho^2}\right) \cdot \frac{z}{(\rho^2+z^2)^{3/2}} - \frac{S' \cdot S}{R_{LC}^2-\rho^2} = 0$$

$$\frac{S'\cdot S}{R_{LC}^2-\rho^2} = -\Psi_0 \cdot \frac{z}{(\rho^2+z^2)^{3/2}} + \Psi_0 \cdot \frac{z}{(\rho^2+z^2)^{3/2}} \cdot \frac{R_{LC}^2+\rho^2}{R_{LC}^2-\rho^2}$$

$$= \Psi_0 \cdot \frac{z}{(\rho^2+z^2)^{3/2}} \cdot \frac{2\rho^2}{R_{LC}^2-\rho^2}$$

$$\Rightarrow S' S = \Psi_0 \cdot \frac{2z\rho^2}{(\rho^2+z^2)^{3/2}} = \Psi \cdot \frac{2\rho^2}{\rho^2+z^2}$$

$$= \Psi \cdot 2 \cdot \left(1 - \frac{\Psi^2}{\Psi_0^2}\right)$$

$$\frac{dS^2}{d\Psi} = \Psi \cdot \left(1 - \frac{\Psi^2}{\Psi_0^2}\right)$$

$$S = A\Psi^2 + B\Psi + C$$

$$S' = 2A\Psi + B$$

$$S \cdot S' = \cancel{2A^2\Psi^3 + 3AB\Psi^2}$$

$$2A^2\Psi^3 + AB\Psi^2 + 2AB\Psi^2 + B^2\Psi + 2AC\Psi + BC$$

$$= 2A^2\Psi^3 + 3AB\Psi^2 + (B^2+2AC)\Psi + BC$$

$$2A^2 = \frac{2}{\Psi_0^2} \qquad \therefore A = \frac{1}{\Psi_0}$$

$$B = 0,$$

$$2AC = -2 \qquad C = -\Psi_0$$

$$\Rightarrow S = \frac{1}{\Psi_0}\Psi^2 - \Psi_0 \quad \checkmark$$

$$\vec{B} = \left(-\Psi_0 \cdot \frac{\rho}{(\rho^2+z^2)^{3/2}},\ \frac{1}{R_{LC}}(\Psi^2 - \Psi_0^2)\frac{1}{\rho},\ -\Psi_0 \cdot \frac{z}{(\rho^2+z^2)^{3/2}}\right)$$

= split monopole

赤道 $\Psi = 0$

$\rho\vec{E} + \frac{1}{c}(\vec{j}\times\vec{B}) = 0$ 　极轴 $\Psi_0$ ($z\to\infty$)

$$\Rightarrow (\nabla\cdot\vec{E})\vec{E} + \left[(\nabla\times\vec{B})\times\vec{B}\right] = 0$$

FFE 　E.o.M & Ohm's Law:

plug in $\vec{B} = \frac{\nabla\Psi\times\vec{e}_\phi}{\rho} + \frac{\Psi_0}{R_{LC}}\cdot\frac{S}{\rho}\vec{e}_\phi$

& $\vec{E} + (\vec{\Omega}\times\vec{r})\times\vec{B} = 0$

$j = \sigma(\vec{E} + \vec{v}\times\vec{B})$, $\sigma\to\infty$, $(\vec{E}+\vec{v}\times\vec{B})\to 0$

$\Rightarrow$ PSR Eq 　$\sigma\to\infty$ 的结果